VOLUME LXIII NUMBER 5

# THE ASTROPHYSICAL JOURNAL

## AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

*Edited by*

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

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## JUNE 1926

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AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME LXIII JUNE 1926 NUMBER 5

# THE K-TERM, RELATIVITY DISPLACEMENTS, AND CONVECTION CURRENTS IN B-TYPE STARS

By SEBASTIAN ALBRECHT

ABSTRACT

An *analysis* is made of the various *factors* contributing to the *K-term* of +4.0 km in *B-type stars. Systematic errors in the normal wave-lengths* employed in the computations would change the radial velocities by about −1.4 km/sec., thus *reducing the K-term to about +2.6 km.* In order that *relativity displacements* might be as great as +2.6 km it would be necessary that the *average mass* of B-type stars be about 25⊙ with a density of 0.1 or about 10⊙ with a density of about 0.6. Although either of these alternatives is possible, the most probable amount of the *relativity shift*, based on the generally accepted values for average mass and density, *is +1.4 km*, in terms of velocity. General *downward convection currents* may be the cause for the remaining *+1.2 km* of the K-term, with about +2.0 km as an upper limit and zero as a lower limit for this effect.

In a suggestive article on "Convection Currents in Stellar Atmospheres," St. John and Adams,[1] extrapolating beyond their data, attempt to explain the K-term in the B-type stars by radial motions of the radiating gases. In their statement (p. 49), "It seems quite reasonable to conclude from these results that at the higher temperatures of the B-type stars the downward velocities of the convection currents will attain a value of the order of 4 km, the amount of the K-term," they entirely ignore the effect of the systematic errors in the normal wave-lengths which were employed in the reductions for radial velocity and the appreciable contribution (of the same sign as the K-term) which must result from the effects of generalized relativity. If we employ data which are available for an approximation of the magnitude of the component of the K-term due to each

[1] *Astrophysical Journal*, 60, 43, 1924.

of these effects,[1] it will be found that such general downward convection currents as may be present in B-type stars in the gases in which the *H*, *He*, *Mg*, *Si*, *O*, and *N* lines originate must be of very moderate magnitude.

The effects upon the K-term of systematic errors in the normal wave-lengths which were employed in the reductions for radial velocity have been discussed in my two previous articles.[2] Although the laboratory wave-lengths needed for this purpose have not been determined with the high degree of accuracy required for a definitive solution of the problem, we can say with a reasonable degree of certainty that for classes Bo to B5 the radial velocities will be changed in the mean by between $-1$ and $-2$ km per second. The probable value indicated for this effect is $-1.4$ km, thus reducing the K-term from $+4$ km to about $+2.6$ km.

Identification of the K-terms as a generalized relativity displacement was, I believe, first proposed by E. Freundlich.[3] However, unless the masses or the densities of B-type stars are very much greater than the generally accepted values, relativity shifts cannot account for the entire K-term. The formula to be employed for the determination of the relativity displacement of the spectrum of any star, as corrected by H. Seeliger,[4] is

$$M=(K'\div 0.634)^{\frac{3}{2}}\cdot\rho^{-\frac{1}{2}}$$

in which $M$ is the mass of the star in terms of the sun's mass,

$K'$ is the relativity displacement of the stellar spectrum, in km per sec.,

$\rho$ is the density of the star in terms of the sun's density,

or, employing the form given by de Sitter,[5]

$$K'=0.634\,M^{\frac{3}{2}}\cdot\rho^{\frac{1}{2}}.$$

[1] St. John and Adams have shown that the pressures in stellar atmospheres are very small. We may, therefore, regard pressure displacements as negligible in this problem.

[2] *Astrophysical Journal*, **55**, 361, 1922, and **57**, 57, 1923.

[3] *Astronomische Nachrichten*, **202**, 17, 1915.

[4] *Ibid.*, **202**, 83, 1916.

[5] *Monthly Notices of Royal Astronomical Society*, **76**, 719, 1916.

With an adopted average mass of 10⊙ and a density of 0.1, Shapley's value, de Sitter derived a relativity displacement equivalent to +1.4 km/sec. This is probably a fair approximation to the actual value of the relativity displacement of the spectrum of B-type stars. However, there is a possibility that it may be larger, perhaps double this amount. The following table gives computed values of the relativity displacement for various assumed values of density and mass. Thus, a K-term of about +2.6 km, which remains to be accounted for,

| Density | Mass | Relativity Displacement (at Wave-Length 4500) | |
|---|---|---|---|
| | | angstroms | km/sec. |
| 0.01 | 10⊙ | +.009 | +0.63 |
| 0.1 | 5 | .013 | 0.86 |
| 0.1 | 10 | .020 | 1.36 |
| 0.1 | 15 | .027 | 1.79 |
| 0.1 | 20 | .033 | 2.17 |
| 0.1 | 30 | .043 | 2.87 |
| 0.1 | 50 | .060 | 4.0 |
| 0.5 | 10 | .035 | 2.3 |
| 0.5 | 20 | .056 | 3.7 |
| 1.0 | 10 | .044 | 2.9 |
| 1.1 | 10⊙ | +.058 | +3.9 |

could be produced by an average mass of 25⊙ with a density of 0.1 or by an average mass of 10⊙ with a density of 0.6. The large average mass of 25⊙, especially for types Bo to B5, is entirely within the range of possibility as will be shown below. Even an average mean density of 0.6, though not in complete harmony with other known facts, is not wholly excluded. In this connection it is well to recall that Eddington has demonstrated that the ionized star can attain to far higher densities than the mass of un-ionized gas.

Jeans[1] has tabulated a list of 30 early type spectroscopic binaries for which either $m_1+m_2$ or $(m_1+m_2)\sin^3 i$ could be determined. With Schlesinger's average value of 0.68 for $\sin^3 i$ for all detected binaries (which involves a slight assumption) Jeans derived an average mass of 19⊙. By subdividing his list, we find these average values for the combined mass:

Mean of 14 values, Bo to B5, 34⊙
Mean of 6 values, B8 to B9, 11⊙
Mean of 10 values, Ao to A5, 4⊙ .

[1] *Ibid.*, **85**, 800, 1925.

The large K-term pertains principally to types Bo to B5, for which the large increase in mass is indicated above. Seeliger has pointed out that for a point on the surface of either component of a spectroscopic double star about two-thirds of the combined mass $(m_1+m_2)$ is the maximum value which is effective in producing the gravitation potential. According to Jeans the average values of the constituents after fission are 0.59 and 0.41 of the combined mass, and almost invariably the larger constituent is the more luminous. Therefore, $0.6\,(m_1+m_2)$ would seem to be a reasonable assumption for the value of the effective mass producing the surface gravitation potential on the component star whose light is recorded on our spectrograms. Applied to the 14 spectroscopic binaries of types Bo to B5 contained in the list by Jeans, we get an effective mass of $20\odot$ which, with Shapley's value of 0.1 for the average density, would give a relativity displacement of +2.2 km/sec. in terms of velocity. This is only slightly below the value of the unexplained portion of the K-term, and would thus leave very little to be accounted for by downward convection currents.

Although such a large average value for the mass of B-type stars is supported by good evidence, it cannot be regarded as definitely established. Thus, O. Struve[1] obtains an average mean mass of $8\odot$ for O- and B-type binaries from a relation between period and semi-amplitude of range of radial velocity. However, in order that general downward convection currents might be as great as 2.0 km/sec., which is one-half of the original value of the K-term, either the average density would have to be reduced to 0.01 or the average mass to about $4\odot$. Present evidence thus indicates 2 km/sec. as an upper limit to general inward convection currents of the absorbing gases in B-type stars, with a fair possibility that they may be practically negligible.

DUDLEY OBSERVATORY
March 1926

[1] *Observatory*, **49**, 6, 1926.

# A SPECTROGRAPHIC STUDY OF MIRA CETI[1]

By ALFRED H. JOY


## ABSTRACT

*Spectral variations.*—131 spectrograms taken with the 60-inch and 100-inch reflectors cover the whole range of the star's variation during the last ten years. Spectral changes are recurrent. The titanium bands vary with magnitude (Table III). Bright lines of ionized iron are seen near maximum. Although at minimum there are no emission lines, bright hydrogen lines appear soon after and have their greatest intensity near maximum. Low-temperature emission lines of iron, magnesium, and silicon appear after maximum is well past. The weaker absorption lines fade out at minimum. Spectral changes depend mostly on the magnitude of the star.

*Absorption spectrum.*—Measures of individual lines of band structure show that the bands give the same velocities as the absorption lines. Seven new titanium oxide bands to the violet of λ 4354 have been identified by laboratory comparisons. The absorption lines belong to temperature classes I and II. The most prominent elements are iron, vanadium, chromium, manganese, calcium, and magnesium. Titanium lines are weak. Radial velocity measures give a curve (Table VI) which is represented by elliptical elements.

*Emission lines.*—The bright lines measured are given in Table VIII; the probable identification of 32 of these lines is in Table X. Estimates of the intensities at different phases have been made for 36 of the stronger lines (Table IX). The emission lines give a velocity curve (Table XIV) which shows outward motion relative to the absorption-line curve except at minimum, when the difference is zero. The maximum difference is 19 km/sec. at phase 56 days. The velocities of both bright and dark lines are the same at recurring cycles (Table XVII). The intensities of the bright lines at maximum depend on the magnitude of the star (Table XIX). At the unusually faint maximum of 1924 several peculiar features were noted in the region λ 4584–λ 4905, which, in part, are tentatively attributed to magnesium hydride.

*Physical conditions.*—The spectroscopic absolute magnitude at the normal maximum of magnitude 3.5 is estimated to be −0.3, corresponding to a parallax of 0".017. From the angular diameter of 0".056 measured with the interferometer, the linear diameter is found to be 490,000,000 km, and the surface brightness 7.5 mags. fainter than the sun. For an assumed mass 5 times that of the sun the density is $1.1\times10^{-7}$, and the surface gravity $4\times10^{-5}$, that of the sun. The temperature is estimated to vary from 2300° to 1800° K. The observed bolometric variation in energy of about one magnitude agrees with the observed variation in visual magnitude, when account is taken of the shift in the energy-curve toward the red for these temperatures, and the excessive absorption of the titanium oxide bands at minimum.

*The visual companion.*—The spectrum of a companion was seen on plates taken near minimum. Its approximate position was determined from the spectrograms. Later its presence was confirmed visually by Aitken. In 1923 the magnitude was 9.8, but it seems to be a variable. It has an early type spectrum with bright lines of hydrogen, helium, and ionized iron and calcium. No absorption lines are present except those connected with emission bands of the hydrogen series. The violet component of the hydrogen lines is affected by the variation of both stars.

*Discussion.*—The application of recent results to the problem of the variation of Mira and similar stars is briefly discussed.


The purpose of this contribution is to give a summarized description of the extraordinary changes which take place in the spectrum

[1] *Contributions from the Mount Wilson Observatory*, No. 311

of the long-period variable *o* Ceti, to extend the material published in several papers[1] by Adams and the writer, and to give the results of further study of the spectrograms of the star made with the 60-inch and 100-inch reflectors during the years 1916–1925.

Mira is the brightest and the best-known star of its class. In most respects its behavior is typical. The long-period variables are distinguished by periods averaging about 300 days. Most of them have M-type spectra with emission lines, particularly of hydrogen.

Since the discovery of its variation in 1596, the light changes in Mira have been followed with very few interruptions. The mean period is 330 days, but the interval from maximum to maximum may vary by as much as 30 days. The magnitude at maximum is normally about 3.5, with extremes of 1.5 and 5.6. The minimum is also uncertain, varying from 8.0 to 10.0, but most frequently is about 9.2. The red color of the variable has made the estimates of its brightness somewhat uncertain, especially at minimum phase.

A number of observers have studied the spectrum of the star at or near maximum. Stebbins[2] in his extended research in 1903 was able to follow it to lower magnitudes, but the faintness of the star at minimum phase made it impossible at that time to secure satisfactory spectrograms.

Observations were begun at Mount Wilson in 1916, primarily to note the effects of changing absolute magnitude on the spectrum. As observations continued, points of astrophysical interest developed which had not previously been considered in full, so that it was deemed advisable to try to cover the whole cycle of variation. Accordingly more time has been devoted to this star than to any other thus far observed spectroscopically at this Observatory.

The completion of the 100-inch reflector in 1919 made it possible to secure spectrograms at minimum with sufficient dispersion to study the velocity variation of bright and dark lines throughout the period, as well as to note numerous other data bearing on the physical conditions in and about the star. The intrusion of a secondary

[1] *Publications of the Astronomical Society of the Pacific*, **29**, 112, 1917; **30**, 193, 1918; **32**, 163, 1920; **33**, 107, 1921; **35**, 168, 1923; **36**, 290, 1924. *Publications of the American Astronomical Society*, twenty-ninth meeting, p. 14, 1922; thirtieth meeting, p. 64, 1923.

[2] *Lick Observatory Bulletins*, **2**, 78, 1903; *Astrophysical Journal*, **18**, 341, 1903.

spectrum at minimum led, in 1920, to the consideration of the existence of a faint companion,[1] and to its visual detection[2] in 1923.

## 1. THE GENERAL COURSE OF THE SPECTRAL VARIATIONS

The major physical processes taking place in the star seem to repeat themselves without essential change in succeeding cycles of its light variation. The outstanding features of the spectrum depend for the most part upon the magnitude of the star as well as upon the phase.

At maximum the star is of spectral type M5–M6, with strong bands of titanium oxide and sharp hydrogen emission lines. The

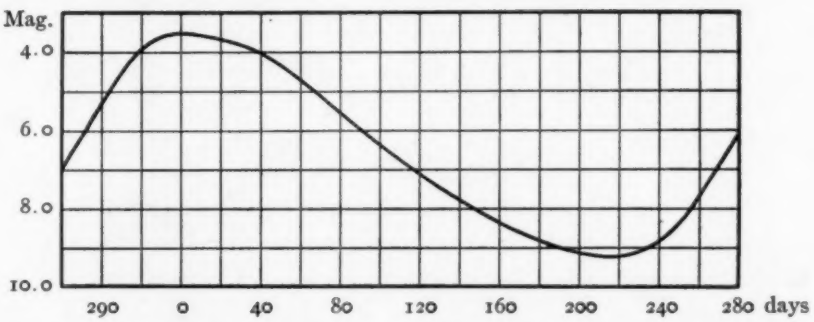



FIG. 1.—Light-curve of Mira from observations of the American Association of Variable Star Observers.

bright lines of hydrogen have their maximum intensity at or soon after the maximum of light. They are accompanied by faint emission lines of ionized iron. The appearance of most of the other bright lines is delayed for three or four months until the star has reached fainter magnitudes.

When the star is fainter than the eighth magnitude the bright lines of the companion are well seen and its continuous spectrum shows sufficient strength to give the combined spectrum a curious distorted effect which is especially noticeable in the region of the bands. As the brightness diminishes the bands increase in strength and the absorption lines fade until at minimum only a few lines can

[1] *Publications of the American Astronomical Society*, twenty-ninth meeting, p. 14, 1922.

[2] Aitken, *Publications of the Astronomical Society of the Pacific*, **35**, 323, 1923.

be seen. Certain lines of chromium, calcium, and vanadium seem to be the most persistent. The strong calcium line at λ 4226 undergoes a tremendous change, reaching a width of about 25 A at minimum.

The sharp emission lines of hydrogen disappear at about magnitude 8.5 and reappear at about the same brightness on the rising branch of the light-curve. There are numerous other emission lines, mostly sharp in character like the hydrogen lines, the time of whose appearance is apparently determined by their temperature class. The silicon lines λλ 3905 and 4103 are seen near maximum, but the extremely low-temperature magnesium line λ 4571 is not found until a magnitude of 5.5 is reached about 80 days after maximum. All bright lines disappear at minimum, or very soon after, so that during an interval of 20–40 days soon after minimum no emission lines whatsoever are given out by the variable itself.

## 2. THE OBSERVATIONS

The observations have extended over a sufficient length of time to be fairly representative of the spectrographic behavior of the star. All portions of the light-curve have been covered. The maxima observed have varied from 2.8 in 1923 to 4.7 in 1924; the extremes of minima were 9.4 in 1921 and 8.5 in 1922.

In general the spectrograms were taken with a single prism and an 18-inch (46 cm) camera at the Cassegrain focus of the 60-inch and 100-inch reflectors. The usual dispersion was about 35 A to the millimeter. In a few cases a 40-inch (102 cm) camera was used near maxima, and cameras of 7 and 10 inches at the fainter minima. When possible the exposures were timed to show the continuous and dark-line spectrum rather than bright lines only. The plates have been made by a number of observers to whom much credit is due for the results obtained. Some of the spectrograms taken at minimum of light during the poor observing conditions of winter were particularly difficult to obtain.

The magnitudes used in this discussion are based on curves generously furnished from the Harvard College Observatory by Professors Bailey and Shapley and on observations published by the American Association of Variable Star Observers. The brighter mag-

nitudes agree closely with the estimates published in the *Astronomische Nachrichten* by Nijland, but those below 7.5 are systematically brighter, chiefly because of the different system of magnitudes for the comparison stars adopted by Nijland.

The dates of observed maxima and minima for the years in question are given in Table I.

TABLE I

MAXIMA AND MINIMA

| MAXIMA | | | | MINIMA | |
|---|---|---|---|---|---|
| Mag. | Observed Date | Calculated Date | O−C | Mag. | Observed Date |
| 3.7 | 1916, Nov. 14 | Nov. 16 | − 2$^d$ | ..... | ..... |
| 3.6 | 1917, Oct. 1 | Oct. 12 | −11 | ..... | ..... |
| 3.5 | 1918, Sept. 20 | Sept. 7 | +13 | ..... | ..... |
| 3.3 | 1919, Aug. 6 | Aug. 3 | + 3 | 9.6 | 1920, Mch. 21 |
| 3.1 | 1920, June 30 | June 28 | + 2 | 9.4 | 1921, Jan. 27 |
| ..... | ..... | May 24 | ..... | 9.5 | 1921, Dec. 28 |
| ..... | ..... | Apr. 19 | ..... | 8.5 | 1922, Nov. 6 |
| 2.8 | 1923, Mch. 6 | Mch. 15 | − 9 | 9.2 | 1923, Oct. 6 |
| 4.7 | 1924, Feb. 1 | Feb. 8 | − 7 | 9.3 | 1924, Sept. 1 |
| 3.8 | 1925, Jan. 4 | Jan. 3 | + 1 | 9.0 | 1925, Aug. 1 |
| 3.3 | 1925, Dec. 4 | Nov. 29 | + 5 | ..... | ..... |

The question of phase in a case of this kind is a difficult problem, and thus far no adequate method has been suggested by which observations made in different cycles can be satisfactorily combined. Some have taken as phases proportional parts of the time actually elapsing between maxima and minima or between one maximum and the next. In this discussion of *o* Ceti, where the irregularities are small as compared with the whole period and where phases are used chiefly in connection with radial velocities which, at best, have a large probable error, the use of a mean period has seemed admissible. All phases given in this paper are based upon maxima determined from the mean elements

$$\text{Max.} = 2421184 + 330 \text{ days.}$$

These elements give the calculated maxima of Table I. It will be observed that the deviations of the observed maxima from the calculated values do not exceed 13 days.

Table II is a list of the 131 spectrograms of *o* Ceti used in this

TABLE II

OBSERVATIONS OF *o* CETI

| | Plate | Date | J.D. | Mag. | Phase | Spectrum | Dark Lines | | Bright Lines | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | *V* | Wt. | *V* | Wt. |
| | | 1916 | | | | | km/sec. | | km/sec. | |
| γ | 5252..... | Nov. 11 | 1179 | 3.7 | 325 | M7 | +63.4 | 1 | +42.8 | 1 |
| | 5258..... | 12 | 1180 | 3.7 | 326 | 7 | 62.4 | 3 | 46.4 | 2 |
| | 5299..... | Dec. 5 | 1203 | 3.8 | 19 | 7 | 64.2 | 2 | 46.3 | 2 |
| | | 1917 | | | | | | | | |
| | 5413..... | Jan. 4 | 1233 | 4.4 | 49 | 7 | 70.3 | 3 | 40.3 | 2 |
| | 5480..... | 31 | 1260 | 5.2 | 76 | 8 | 58.2 | 3 | 40.7 | 2 |
| | 5492..... | Feb. 1 | 1261 | 5.3 | 77 | 8 | 58.5 | 3 | 40.3 | 2 |
| | 5559..... | 9 | 1269 | 5.5 | 85 | 8 | 66.2 | 2 | 46.4 | 2 |
| | 5615..... | Mch. 3 | 1291 | 6.2 | 107 | 8 | 54.5 | 2 | 39.5 | 2 |
| | 6120..... | Aug. 27 | 1468 | 5.9 | 284 | 8 | 63.5 | 2 | 47.4 | 2 |
| | 6235..... | Sept. 30 | 1502 | 3.6 | 318 | 6 | 65.4 | 2 | 49.7 | 2 |
| | 6236..... | 30 | 1502 | 3.6 | 318 | 6 | 64.1 | 2 | 46.0 | 2 |
| | 6243..... | Oct. 1 | 1503 | 3.6 | 319 | 6 | 62.7 | 3 | 51.5 | 1 |
| | 6270..... | 5 | 1507 | 3.6 | 323 | 6 | 64.6 | 2 | 45.2 | 2 |
| | 6306..... | 26 | 1528 | 3.7 | 14 | 7 | 63.8 | 3 | 49.2 | 2 |
| | 6338..... | Nov. 1 | 1534 | 3.8 | 20 | 7 | 63.4 | 3 | 49.3 | 1 |
| | 6369..... | 23 | 1556 | 4.1 | 42 | 7 | 69.7 | 2 | 48.3 | 2 |
| | | 1918 | | | | | | | | |
| | 6531..... | Jan. 2 | 1596 | 5.1 | 82 | 7 | 60.0 | 2 | 40.6 | 3 |
| | 6594..... | 23 | 1617 | 5.7 | 103 | 8 | 64.1 | 3 | 40.2 | 3 |
| | 6724..... | Mch. 2 | 1655 | 7.1 | 141 | 8+Bβ | 48.7 | 1 | 38.3 | 2 |
| | 7413..... | Sept. 23 | 1860 | 3.5 | 16 | 6 | 59.1 | 2 | 48.0 | 2 |
| | 7524..... | Oct. 26 | 1893 | 3.9 | 49 | 6 | ..... | .... | ..... | .... |
| | 7551..... | Nov. 16 | 1914 | 5.2 | 70 | 7 | 68.5 | 2 | 49.1 | 2 |
| | 7645..... | Dec. 15 | 1943 | 6.5 | 99 | 8 | ..... | .... | 45.2 | 3 |
| | | 1919 | | | | | | | | |
| | 7773..... | Jan. 16 | 1975 | 7.7 | 131 | e | ..... | .... | 48.4 | 1 |
| | 7795..... | 18 | 1977 | 7.8 | 133 | 8+Bβ | 53.4 | 1 | 46.3 | 3 |
| | 8597..... | Aug. 16 | 2187 | 3.4 | 13 | 5 | 65.8 | 3 | 44.3 | 2 |
| | 8694..... | Sept. 10 | 2212 | 3.8 | 38 | 6 | 56.2 | 2 | 40.8 | 1 |
| | 8789..... | Oct. 14 | 2246 | 5.3 | 72 | 7 | 60.1 | 2 | 41.6 | 3 |
| | 8805..... | Nov. 3 | 2266 | 6.2 | 92 | 8 | 56.9 | 3 | 38.9 | 3 |
| C | 185..... | 8 | 2271 | 6.4 | 97 | ..... | ..... | .... | ..... | .... |
| γ | 8872..... | 13 | 2276 | 6.6 | 102 | 8 | ..... | .... | 45.8 | 3 |
| | 8918..... | Dec. 12 | 2305 | 7.7 | 131 | e+Bβ | ..... | .... | 53.3 | 3 |
| | | 1920 | | | | | | | | |
| C | 230..... | Jan. 7 | 2331 | 8.4 | 157 | e+Bβ | ..... | .... | 51.8 | 2 |
| | 246..... | 14 | 2338 | 8.5 | 164 | e+Bγ | ..... | .... | 55.3 | 2 |
| | 250..... | 15 | 2339 | 8.6 | 165 | 9+Bδ | ..... | .... | 56.6 | 3 |
| | 260..... | 29 | 2353 | 8.9 | 179 | 9+Bδ | ..... | .... | 55.4 | 2 |
| | 278..... | Feb. 6 | 2361 | 9.1 | 187 | e+Bβ | ..... | .... | 57.2 | 1 |
| | 287..... | 12 | 2367 | 9.2 | 193 | e+Bδ | ..... | .... | 54.0 | 1 |
| | 500..... | June 29 | 2505 | 3.1 | 1 | 6 | 65.0 | 3 | 56.4 | 2 |
| γ | 9395..... | July 27 | 2533 | 3.4 | 29 | 5 | 64.6 | 3 | 47.6 | 2 |
| | 9436..... | Aug. 5 | 2542 | 3.6 | 38 | 5 | 60.1 | 3 | 42.4 | 2 |
| | 9454..... | 23 | 2560 | 4.4 | 56 | 7 | 61.5 | 3 | 37.8 | 2 |
| | 9567..... | Sept. 22 | 2590 | 6.0 | 86 | 8 | 59.5 | 1 | ..... | .... |
| C | 760..... | Nov. 20 | 2649 | 8.1 | 145 | 9+Bγ | 67.5 | 0.5 | 50.2 | 3 |
| | 766..... | 21 | 2650 | 8.2 | 146 | 9+Bδ | 46.3 | 1 | 49.2 | 3 |
| | 801..... | 29 | 2658 | 8.4 | 154 | 9+Bγ | +46.1 | 0.5 | +48.5 | 2 |

TABLE II—*Continued*

| Plate | Date | J.D. | Mag. | Phase | Spectrum | Dark Lines | | Bright Lines | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | *V* | Wt. | *V* | Wt. |
| | 1921 | | | | | km/sec. | | km/sec. | |
| C 847..... | Jan. 4 | 2694 | 9.2 | 190 | M9+Bδ | ..... | .... | +47.6 | 2 |
| 859..... | 21 | 2711 | 9.4 | 207 | −+Bγ | ..... | .... | ..... | .... |
| 861..... | 24 | 2714 | 9.4 | 210 | −+Bβ | ..... | .... | ..... | .... |
| 867..... | 29 | 2719 | 9.4 | 215 | 9+Bε | +47.5 | 0.5 | ..... | .... |
| 874..... | Feb. 15 | 2736 | 9.3 | 232 | 9+Bγ | ..... | .... | ..... | .... |
| 878..... | 17 | 2738 | 9.3 | 234 | 9+Bγ | 49.1 | 0.5 | ..... | .... |
| γ 10276..... | July 14 | 2885 | 4.9 | 51 | 7 | 57.1 | 3 | 40.5 | 2 |
| 10381..... | Aug. 18 | 2920 | 6.4 | 86 | 8 | 54.5 | 2 | 37.8 | 3 |
| 10435..... | Sept. 10 | 2943 | 7.2 | 109 | 8 | 59.8 | 2 | 51.8 | 3 |
| 10503..... | 21 | 2954 | 7.6 | 120 | 8+Bβ | 56.1 | 3 | 44.0 | 3 |
| C 1364..... | Oct. 6 | 2969 | 8.0 | 135 | 8 | ..... | .... | 49.6 | 3 |
| 1375..... | 8 | 2971 | 8.1 | 137 | 8+Bδ | 52.7 | 1 | 45.9 | 3 |
| 1401..... | 13 | 2976 | 8.3 | 142 | ..... | ..... | .... | ..... | .... |
| 1442..... | Nov. 7 | 3001 | 8.8 | 167 | 9+Bδ | 58.0 | 2 | 54.4 | 3 |
| 1474..... | 22 | 3016 | 9.0 | 182 | 9+Bγ | ..... | .... | 45.8 | 1 |
| 1478..... | Dec. 7 | 3031 | 9.2 | 197 | 9+Bδ | ..... | .... | 48.6 | 2 |
| 1482..... | 8 | 3032 | 9.3 | 198 | −+Bβ | ..... | .... | 49.4 | 1 |
| 1483..... | 9 | 3033 | 9.3 | 199 | −+Bβ | ..... | .... | 49.6 | 1 |
| 1488..... | 11 | 3035 | 9.3 | 201 | −+Bγ | ..... | .... | 59.6 | 1 |
| 1494..... | 14 | 3038 | 9.4 | 204 | 9+Bδ | ..... | .... | 50.6 | 1 |
| | 1922 | | | | | | | | |
| 1515..... | Jan. 9 | 3064 | 9.3 | 230 | e+Bγ | ..... | .... | ..... | .... |
| 1546..... | Feb. 4 | 3090 | 8.9 | 256 | 9+Bγ | ..... | .... | ..... | .... |
| 1547..... | 5 | 3091 | 8.9 | 257 | 9+Bδ | 49.4 | 1 | ..... | .... |
| 1597..... | Mch. 9 | 3123 | 7.1 | 289 | 8+Bγ | 60.2 | 1 | ..... | .... |
| γ 11148..... | July 6 | 3242 | 5.7 | 78 | 8 | 58.2 | 2 | 46.0 | 2 |
| 11254..... | Aug. 11 | 3278 | 6.8 | 114 | 8 | 59.5 | 3 | 40.0 | 3 |
| C 1845..... | 31 | 3298 | 7.5 | 134 | 9+Bβ | 57.2 | 2 | 49.8 | 3 |
| 1897..... | Sept. 28 | 3326 | 8.2 | 162 | 9+Bγ | 63.7 | 0.5 | 47.2 | 3 |
| 1950..... | Oct. 9 | 3337 | 8.3 | 173 | ..... | ..... | .... | ..... | .... |
| 1996..... | Nov. 5 | 3364 | 8.5 | 200 | 9+Bδ | 46.2 | 2 | ..... | .... |
| 2013..... | Dec. 5 | 3394 | 8.3 | 230 | 9+Bδ | 53.5 | 3 | ..... | .... |
| 2017..... | 26 | 3415 | 7.9 | 251 | 9+Bγ | 56.3 | 3 | ..... | .... |
| | 1923 | | | | | | | | |
| 2040..... | Jan. 2 | 3422 | 7.8 | 258 | 9+Bγ | 55.7 | 2 | ..... | .... |
| 2041..... | 2 | 3422 | 7.8 | 258 | 8+Bγ | 65.6 | 0.5 | ..... | .... |
| γ 11564..... | 25 | 3445 | 5.5 | 281 | 8 | 58.8 | 3 | 43.2 | 1 |
| C 2090..... | 27 | 3447 | 5.0 | 283 | 7 | 62.3 | 3 | 55.6 | 1 |
| 2091..... | 27 | 3447 | 5.0 | 283 | 7 | 60.9 | 3 | 52.0 | 1 |
| γ 11609..... | Feb. 7 | 3458 | 3.8 | 294 | 6 | 58.7 | 3 | 44.3 | 1 |
| 11622..... | 24 | 3475 | 2.9 | 311 | 5 | 64.3 | 3 | 41.1 | 1 |
| 11623..... | 24 | 3475 | 2.9 | 311 | 5 | 69.2 | 2 | 49.3 | 1 |
| C 2171..... | Mch. 28 | 3507 | 3.2 | 13 | 5 | 65.1 | 2 | 48.1 | 1 |
| 2353..... | July 24 | 3625 | 7.7 | 131 | 8+Bε | ..... | .... | 51.0 | 1 |
| 2383..... | Aug. 18 | 3650 | 8.4 | 156 | 8+Bε | 58.9 | 0.5 | 57.7 | 2 |
| 2482..... | Oct. 19 | 3712 | 9.2 | 218 | 9+Bδ | ..... | .... | +56.3 | 1 |
| 2487..... | Oct. 20 | 3713 | 9.2 | 219 | ..... | ..... | .... | ..... | .... |
| 2579..... | Dec. 17 | 3771 | 8.1 | 277 | 8+Bδ | +53.0 | 2 | ..... | .... |

TABLE II—*Continued*

| Plate | Date | J.D. | Mag. | Phase | Spectrum | Dark Lines | | Bright Lines | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | *V* | Wt. | *V* | Wt. |
| | 1924 | | | | | km/sec. | | km/sec. | |
| γ 12363..... | Jan. 12 | 3797 | 5.6 | 303 | M7 | +61.7 | 2 | +51.0 | 0.5 |
| 12447..... | 26 | 3811 | 5.1 | 317 | 7 | 57.7 | 1 | 41.8 | 0.5 |
| C 2663..... | Feb. 11 | 3827 | 4.7 | 3 | 7 | 64.5 | 2 | 49.9 | 1 |
| γ 12461..... | 13 | 3829 | 4.8 | 5 | 7 | 63.9 | 3 | 43.2 | 1 |
| 12471..... | 14 | 3830 | 4.8 | 6 | 7 | ..... | .... | 45.6 | 1 |
| C 2710..... | Mch. 11 | 3856 | 5.4 | 32 | 7 | ..... | .... | 47.1 | 1 |
| 2930..... | Aug. 12 | 4010 | 9.0 | 186 | −+Bε | ..... | .... | 51.4 | 0.5 |
| 2934..... | 13 | 4011 | 9.0 | 187 | 9 | ..... | .... | 54.3 | 1 |
| 2989..... | Sept. 7 | 4036 | 9.1 | 212 | −+Bδ | ..... | .... | 47.6 | 1 |
| 3037..... | Oct. 8 | 4067 | 8.9 | 243 | 9+Bδ | 55.8 | 1 | 50.2 | 1 |
| 3068..... | Nov. 6 | 4096 | 8.4 | 272 | 9+Bδ | 61.3 | 3 | 44.8 | 0.5 |
| 3078..... | 14 | 4104 | 7.6 | 280 | 8+Bδ | 61.4 | 3 | ..... | .... |
| 3091..... | Dec. 3 | 4123 | 5.2 | 299 | 8 | 55.9 | 2 | 51.6 | 1 |
| | 1925 | | | | | | | | |
| G 47..... | Jan. 3 | 4154 | 3.8 | 0 | 7 | ..... | .... | ..... | .... |
| γ 13185..... | 8 | 4159 | 3.8 | 5 | 7 | 61.3 | 5 | ..... | .... |
| 13191..... | 10 | 4161 | 3.8 | 7 | 7 | ..... | .... | ..... | .... |
| 13192..... | 10 | 4161 | 3.8 | 7 | 7 | ..... | .... | ..... | .... |
| 13193..... | 10 | 4161 | 3.8 | 7 | 7 | ..... | .... | ..... | .... |
| 13200..... | 11 | 4162 | 3.8 | 8 | 7 | 67.5 | 2 | 48.2 | 1 |
| 13201..... | 11 | 4162 | 3.8 | 8 | 7 | 69.9 | 3 | 48.6 | 1 |
| 13202..... | 11 | 4162 | 3.8 | 8 | 7 | 61.7 | 3 | 47.8 | 1 |
| 13203..... | 11 | 4162 | 3.8 | 8 | 7 | ..... | .... | +41.9 | 0.5 |
| C 3406..... | July 29 | 4361 | 9.0 | 207 | 9+Bδ | ..... | .... | ..... | .... |
| 3410..... | 30 | 4362 | 9.0 | 208 | 9+Bδ | ..... | .... | ..... | .... |
| 3511..... | Sept. 8 | 4402 | 8.8 | 248 | 9+Bβ | 57.8 | 2 | ..... | .... |
| 3516..... | 23 | 4417 | 8.0 | 263 | 9 | ..... | .... | ..... | .... |
| γ 13819..... | Oct. 7 | 4431 | 6.4 | 277 | 8 | 61.4 | 3 | ..... | .... |
| 13828..... | 8 | 4432 | 6.3 | 278 | 8 | 61.5 | 2 | ..... | .... |
| 13849..... | 26 | 4450 | 4.5 | 296 | 7 | +61.9 | 3 | ..... | .... |
| 13891..... | Nov. 25 | 4480 | 3.4 | 326 | 6 | ..... | .... | ..... | .... |
| 13892..... | 25 | 4480 | 3.4 | 326 | 6 | ..... | .... | ..... | .... |
| C 3604..... | Dec. 7 | 4492 | 3.2 | 8 | 6 | ..... | .... | ..... | .... |
| 3605..... | 7 | 4492 | 3.2 | 8 | 6 | ..... | .... | ..... | .... |
| γ 13925..... | 22 | 4507 | 3.2 | 23 | 6 | ..... | .... | ..... | .... |
| 13926..... | 22 | 4507 | 3.2 | 23 | 6 | ..... | .... | ..... | .... |
| 13928..... | 23 | 4508 | 3.2 | 24 | 6 | ..... | .... | ..... | .... |
| 13931..... | 24 | 4511 | 3.2 | 25 | 6 | ..... | .... | ..... | .... |
| 13933..... | 25 | 4512 | 3.3 | 26 | 6 | ..... | .... | ..... | .... |
| | 1926 | | | | | | | | |
| 13990..... | Jan. 21 | 4539 | 4.0 | 53 | 7 | ..... | .... | ..... | .... |

NOTES TO TABLE II

Plates γ 6243, 6338, and 9454 were taken with the 40-inch camera; γ 5252, 7524, C 867, 874, 878, 1546, 1547, 1597, 2041, 2353, and 2383 with the 7-inch camera; C 2487, 2930, 2934, 3037, 3068, 3078, 3410, 3511, and 3516 with the 10-inch camera. C 185, 1401, 1950, 2487, 3605, γ 13191, 13192, 13193, and G 47 show the red region of the spectrum. γ 13185 was taken by Mr. Sanford with 3 prisms and 18-inch camera; γ 13891, 13892,

NOTES TO TABLE II—*Continued*

13925, 13926, 13928, 13931, and 13990 were taken with 3 prisms and 40-inch camera to study the structure of $H\gamma$ and $H\delta$. G 47 was taken by Mr. Merrill with a grating spectrograph attached to the 100-inch reflector. $\gamma$ 7645, 7773, 8872, 8918, C 230, 246, 801, 859, 861, 874, 1364, 1482, 1483, 1488, 1515 are too weak to show much of the continuous spectrum.

The slit was set at 45° position angle for C 1482, 1483, 1488, 1494, 2017, and at 135° for C 1474, 2013, 2040, 2041.

The exposure times vary from a few minutes up to four hours. The number of plates taken by various observers is: Hubble, 1; Duncan, 2; Humason, 2; Hoge, 5; Strömberg, 5; Sanford, 6; Merrill, 10; Adams, 31; Joy, 69.

study. The sixth column gives the estimated spectral type for each date. The classification used is based on the recommendation of the International Astronomical Union in 1922, which gives *o* Ceti at normal maximum as an example of type M6e, and R Leonis of type M8e. This system permits the inclusion of all the Me stars thus far observed, at minimum as well as maximum, within the subdivisions 0 to 10. Obviously it is a great advanta to be able to classify all the spectra showing well-developed titanium oxide bands in one spectral class. The typical stars used here in connection with this classification were chosen by Mr. Merrill and the writer and are given in *Contribution* No. 264, *Astrophysical Journal*, **58**, 241, 1923. The spectrum of the companion is B and the appended Greek letter shown in the sixth column of the table is that of the hydrogen bright line farthest to the violet seen in the spectrum of the companion.

The measured radial velocities of dark and bright lines with their corresponding weights are given in the last columns of the table.

### 3. SPECTRAL TYPE

A casual inspection of the plates taken at different phases shows, as has been noted by several observers, a considerable variation in type, which, in general, appears to follow the change in brightness. This relationship is indicated in Table III and Figure 2.

If the spectral type thus determined is a function of temperature, the relationship is important and indicates an actual physical change in temperature in the region where the bands and absorption lines originate.

While the general characteristics of the spectra certainly depend

mostly on magnitude, there is no escape from the conclusion that many peculiarities of individual lines or regions vary in different cycles. Thus, the spectra at the bright minimum of 1922 and the faint maxima of February, 1924, without doubt, were somewhat different from the usual spectra at corresponding magnitudes.

TABLE III

RELATIONSHIP OF TYPE AND MAGNITUDE

| Mean Type | Mean Mag. | No. Plates |
|---|---|---|
| M5.2 | 3.1 | 6 |
| M6.3 | 3.7 | 13 |
| M6.9 | 4.6 | 8 |
| M7.6 | 5.5 | 11 |
| M8.0 | 6.4 | 8 |
| M8.4 | 7.7 | 15 |
| M9.0 | 8.9 | 26 |

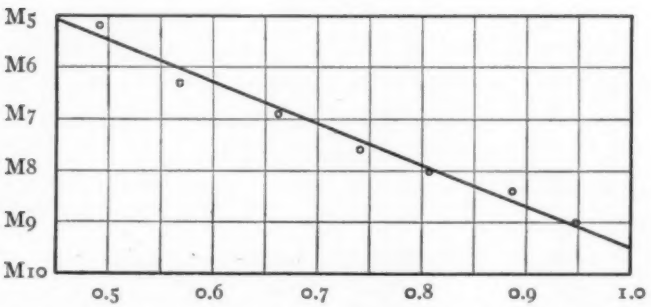


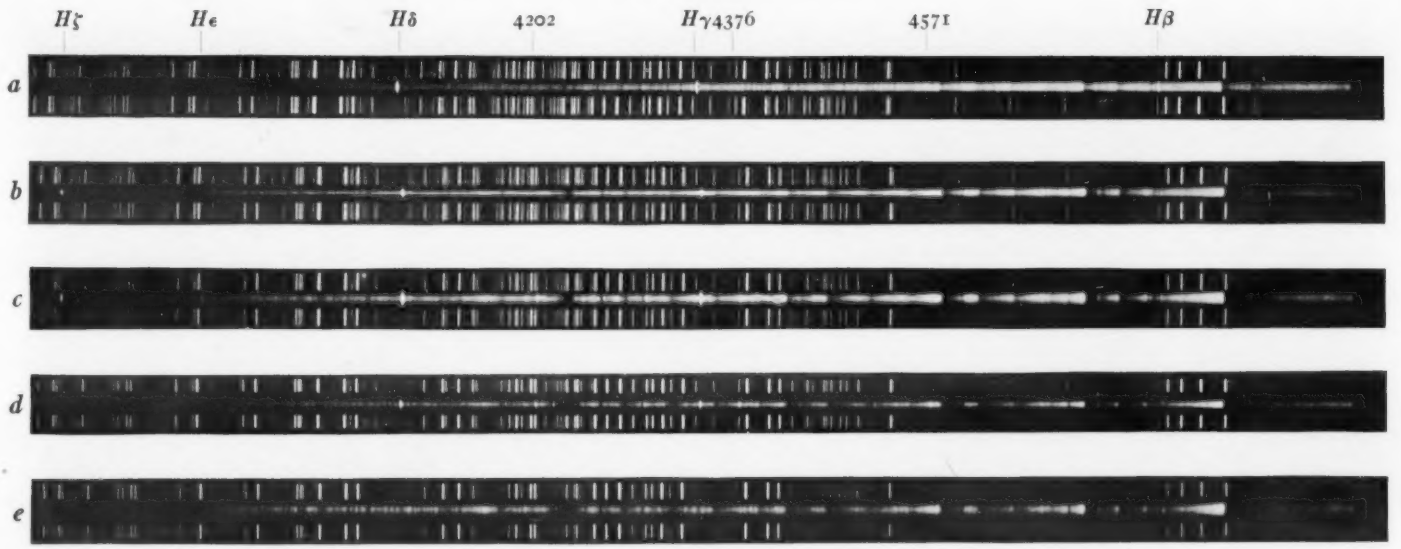

FIG. 2.—Variation of spectral type with logarithm of the magnitude (abscissae)

## 4. THE CONTINUOUS SPECTRUM AND BANDS

With slit spectrograms it is not possible to study satisfactorily the relative intensities of different portions of the continuous spectrum. Nevertheless, it is evident, as Stebbins[1] has shown, that the spectrum is relatively stronger in the region λ 4000–λ 4200 at minimum than at maximum. Mr. Merrill finds the same effect in other Me stars. Doubtless a similar result is indicated by studies of color indices of M-type stars, from which it appears that the color index increases very little, if any, after passing Mo. It is also ap-

[1] *Lick Observatory Bulletins*, **2**, 90, 1903.

# PLATE XVIII




SPECTROGRAMS OF MIRA ILLUSTRATING SPECTRAL VARIATION WITH MAGNITUDE AND PHASE

(*a*) γ 8597, August 16, 1919; magnitude 3.4; type M5e; phase 13 days
(*b*) γ 5413, January 4, 1917; magnitude 4.4; type M7e; phase 49 days
(*c*) γ 6594, January 23, 1918; magnitude 5.1; type M8e; phase 82 days
(*d*) C 1845, August 31, 1922; magnitude 7.5; type M9e; phase 134 days
(*e*) C 2017, December 26, 1922; magnitude 7.9; type M9; phase 250 days

parent that the region to the red of λ 6200 is relatively strong as compared with the yellow and green regions. The relative intensities in the visual region at different phases are well shown in Shane's[1] curves.

The data are not sufficient for a quantitative interpretation of these changes in the continuous spectrum. It is only necessary to point out that two effects are certainly present. The spectrum falls off toward the violet as the energy shifts toward the red with decreasing temperature, and there is a large amount of absorption arising from the greatly increased strength and extent of the titanium oxide bands. These two causes appear to work together near minimum to produce the relatively weak continuous spectrum in the region λ 4200–λ 6200. Some general absorption may be present, but this cannot be proved without better photometric measures of the spectrum than are now available. The subject will be referred to again in Section 12 in connection with a discussion of the temperatures.

The predominating feature of the spectrum of *o* Ceti and the later-type long-period variable stars is the presence of the strong absorption bands identified by Fowler[2] with titanium oxide. The strengthening of these bands in the green and yellow regions of the spectrum plays a considerable part in the light-curve as determined from visual observations. Even if no other causes produced changes in magnitude, the light of the star would still be greatly cut down by the increasing absorption of the titanium oxide. Figure 3 illustrates the strength of the bands near $H\beta$. The figure is taken from Koch microphotometer curves of plate γ 7795. The type is estimated as M8. The positions of these titanium bands in the photographic region have been determined by Stebbins, Plaskett, Sidgreaves, and others, and in the red region of the spectrum by Merrill[3] and Shane.[4] It has been usual to measure the position of the heads of these bands, and from such measures Frost and Lowater[5] have suspected that the bands give the same velocity as the emission lines; but the results have not been satisfactory, because photographic spreading makes

[1] *Ibid.*, **10**, 133, 1922.

[2] *Proceedings of the Royal Society*, A, **79**, 509, 1907.

[3] *Scientific Papers, Bureau of Standards*, No. 318, 1918.

[4] *Lick Observatory Bulletins*, **10**, 132, 1922.

[5] *Astrophysical Journal*, **58**, 272, 1923.

it impossible with the dispersion usually employed in stellar observations to set accurately upon the head of the band. Measures by different observers show large discordances. Settings are not likely to be made in the same manner on plates of different density or upon different bands on the same plate. For these reasons it has not been thought worth while to attempt to measure the heads of the bands.

It was found possible, however, to measure a certain group of lines in the structure of one of the bands of *o* Ceti. The same structural details were identified on a plate of *α* Herculis. Plates *γ* 6243

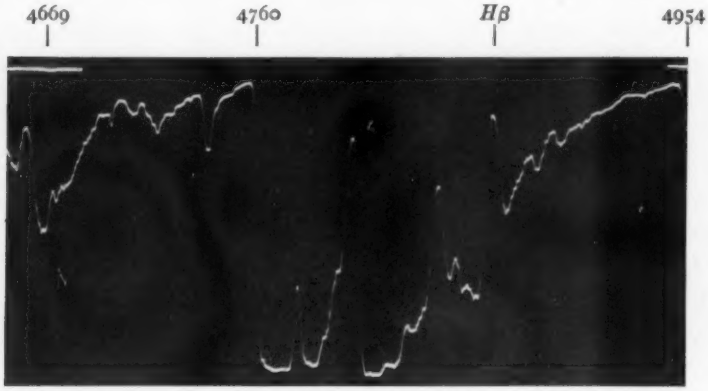



FIG. 3.—Koch microphotometer curve showing the absorption of titanium oxide bands from λλ 4650 to 4955. Spectral type M8.

and *γ* 6338, taken with the 40-inch camera, and *γ* 13185, with 3 prisms and 18-inch camera, show especially sharp detail in the region λ 4609–λ 4623, which is reproduced in Plate XX *e*. The dispersion at this point is 20 A per millimeter for the first two plates, and 17 A per millimeter for the last plate. The lines used are blended pairs and triplets in the tail of a band whose head is at λ 4585. Although many such could have been measured, a series of twelve blends was thought sufficient for the investigation of the velocity given by the bands. The lines are beautifully defined on these plates and settings can be made with the utmost precision. The groups were identified and their wave-lengths determined from two different furnace plates kindly lent by Mr. King, using strontium and titanium lines as standards.

The results of the measurements, on the Rowland system, are given in Table IV. The wave-lengths measured in *o* Ceti have been corrected by the amount of the velocity found from the absorption lines of the spectrum. The results show conclusively that the bands give the same displacements as the absorption lines of the star and indicate that the band absorption originates in the star's

TABLE IV

COMPARISON OF $TiO_2$ IN *o* CETI AND IN THE LABORATORY

| Laboratory Mean λ (2 plates) | *o* Ceti Mean λ (3 plates) | Difference *o* Ceti − Laboratory |
|---|---|---|
| 4608.179 | 4608.090 | −0.089 |
| 4609.247 | 4609.152 | − .095 |
| 4610.199 | 4610.199 | .000 |
| 4611.337 | 4611.430 | + .093 |
| 4612.478 | 4612.490 | + .012 |
| 4613.624 | 4613.665 | + .041 |
| 4614.922 | 4614.955 | + .033 |
| 4616.090 | 4616.124 | + .034 |
| 4617.288 | 4617.344 | + .056 |
| 4618.593 | 4618.570 | − .023 |
| 4619.891 | 4619.839 | − .052 |
| 4621.102 | 4621.106 | + .004 |
| Mean difference | | +0.001 |

atmosphere under conditions similar to those producing the atomic absorption lines.

Several hitherto unrecorded bands appear near the usual minima, and also at the peculiar maximum of 1924. These are degraded toward the red, but the heads are not strongly marked. Their approximate positions are:

| | | |
|---|---|---|
| λλ 4130–50 | λλ 4242–44* | λλ 4315–25 |
| 4165–90 | 4250–68 | 4328–34* |
| 4212–38 | 4270–78 | 4338–48 |

All except the two which are starred have been identified as belonging to the titanium oxide band-spectrum. Plates for this purpose, taken with the furnace, into which had been introduced a considerable current of oxygen, were kindly supplied by Mr. King. Koch microphotometer curves assisted greatly in studying the structure of the bands. One of the most prominent of these new bands is

that at λλ 4212–4238 which is of especial interest because it falls upon the great absorption region due to λ 4226 of calcium and is responsible for a part of the absorption heretofore attributed to neutral calcium.

## 5. ABSORPTION SPECTRUM

The dispersion used in the observations of *o* Ceti does not permit of a detailed study of a large number of lines at different phases of the star's light, but it does give a fair determination of the velocity and makes it possible to observe the behavior of the outstanding lines of the spectrum.

The spectrum at maximum, as has been pointed out by previous observers, contains a large number of absorption lines, especially in the region λ 3900–λ 4350, where the bands are less intense. The identification of these lines shows that those which are measurable with the dispersion used are, with very few exceptions, low-temperature lines of classes I and II belonging to the elements iron, vanadium, chromium, manganese, calcium, and magnesium. Mr. Merrill has called my attention to the presence of the pair λλ 4044 and 4047 of neutral potassium which is strengthened at fainter magnitudes.

The lines of vanadium are remarkable for their number and persistence. They are the only lines of moderate intensity which can be measured near the minimum. The weakness of titanium is striking. The explanation must be along the line of King's experiment.[1] He found that, when the bands of titanium oxide were strengthened by the introduction of oxygen into the furnace, the usual titanium lines practically disappeared. The supply of atomic titanium seems largely to be exhausted by the formation of the $TiO_2$ molecules which produce the bands. The presence of a considerable amount of oxygen is necessary to obtain this result.

The only lines of titanium actually measured on our plates with ordinary dispersion are λ 4314 and the stronger members of the low-temperature multiplets at λ 4300 and λ 4535, and these are notably weakened as the bands strengthen. Several other titanium lines appear faintly at maximum if higher dispersion is employed.

The absorption lines generally used in the velocity measures are given in Table V. The identifications and wave-lengths are given

[1] *Mt. Wilson Contr.*, No. 114; *Astrophysical Journal*, 43, 342, 1916.

in the Rowland and International systems. The temperature class is that found by King in the electric furnace. Additional lines of less intensity are visible on the plates taken with greater dispersion, such

TABLE V

ABSORPTION LINES

| ELEMENT | TEMP. CLASS | WAVE-LENGTH | | ESTIMATED INTENSITY | |
|---|---|---|---|---|---|
| | | Rowland | I.A. | Max. Mag. | Mag. 8.5 |
| *Fe*..... | I A | λ 4291.630 | .473 | 4 | e 2 |
| | I | 4144.038 | .873 | 2 | 1 |
| | | 4202.198 | .033 | 2 | e 14 |
| | | 4376.107 | .934 | 4 | e 14 |
| | | 4427.482 | .314 | 2 | e 2 |
| | II | 4005.408 | .247 | 4 | 1 |
| | | 4045.975 | .816 | 10 | 2 |
| | | 4063.759 | .598 | 5 | 3 |
| | | 4071.908 | .742 | 3 | 2 |
| | | 4250.626 | .471 | 4 | 1 |
| | | 4271.760 | .606 | 5 | 2 |
| | | 4294.301 | .130 | 3 | 1 |
| | | 4308.081 | .908 | 4 | e 15 |
| | | 4325.939 | .766 | 8 | 1 |
| | | 4383.720 | .550 | 6 | 1 |
| | | 4404.927 | .755 | 5 | 1 |
| | | 4415.293 | .128 | 2 | 1 |
| | III | 4260.640 | .482 | 1 | 1 |
| | | 4282.565 | .408 | 2 | 2 |
| | | 4408.582 | .420 | 4 | 3 |
| *V*..... | I | 4092.821 | .671 | 5 | 4 |
| | | 4109.905 | .753 | 3 | 3 |
| | | 4115.330 | .178 | 2 | 2 |
| | | 4116.634 | .482 | 4 | 4 |
| | | 4128.251 | .098 | 3 | 3 |
| | II | 4379.396 | .239 | 4 | 4 |
| | | 4390.149 | .992 | 4 | 3 |
| | | 4395.413 | .255 | 6 | 6 |
| *Cr*..... | I | 4497.023 | .859 | 6 | 6 |
| | | 4254.505 | .350 | 12 | 25 |
| | | 4274.958 | .804 | 10 | 20 |
| *Mn*..... | I | 4030.918 | .773 | 8 | 0 |
| | | 4033.224 | .079 | 5 | 4 |
| | | 4034.644 | .499 | 4 | 2 |
| *Ca*..... | I | 4226.904 | .748 | (3A) | (25A) |
| | | 4318.817 | .658 | 2 | 5 |
| *Ca*..... *Cr*..... | I, II | 4289.725 | .569 | 12 | 25 |
| *Ti*..... | II | 4299.410 | .242 | 4 | 2 |
| | | 4314.964 | .805 | 8 | 6 |
| *Sr*+..... | II | 4077.885 | .735 | 20 | 12 |
| | | 4215.703 | .546 | 15 | 4: |
| *Sr*..... | I | 4607.510 | .337 | 5 | 20 |
| *Mg*..... | I | 4571.275 | .105 | 4 | e 27 |
| *Ba*+..... | II | 4554.211 | .044 | 4 | 1 |

as those of *Co*, *Cr*, *Fe*, *Ti*, and *V* belonging entirely to classes I and II. The intensity of the lines is estimated for maximum and for magnitude about 8.5. At minimum the continuous spectrum of the companion makes estimates too uncertain to be of value. As the star declines in brightness the fainter lines of the various elements tend to disappear. Several of the lines become emission lines and are marked with the letter *e*.

The multiplet group of titanium lines of class II at λ 4535 blends to form a strong line at maximum, which weakens and becomes emission in character as the star approaches minimum. λλ 4299.4 and 4314.9 of titanium behave in much the same way. These lines are probably affected by the shortage of titanium caused by the strengthening of the bands, as well as by the general weakening of absorption lines of the lowest temperature classes.

The ultimate lines of ionized calcium, barium, and strontium are very strong at maximum but are much weakened toward minimum. H and K of calcium are not easily photographed in *o* Ceti because of the weakness of the spectrum in the violet and because of the interference of the spectrum of the companion at minimum. On the few plates having sufficient exposure at this phase, the H and K lines appear to be present but decidedly weakened. The encroachment of the bright line λ 4216 may hasten the disappearance of the strontium line λ 4215 at about magnitude 7.5. When the star is fainter than the eighth magnitude, this region becomes engulfed in the tremendous absorption of the calcium line λ 4226 whose width at minimum light is fully 30 A.

Several enhanced lines of iron and titanium, such as λλ 4233, 4352, 4395, 4417, 4501, 4549, and 4584, are found in the spectrum of *α* Orionis, but the reduced temperature in *o* Ceti, even at maximum, and in stars of the later M-types, makes their appearance weak or doubtful. At minimum light they seem to be entirely lacking in the variable star.

The chromium lines λλ 4254 and 4274 and the calcium line λ 4226 strengthen with decreasing temperature and become enormously strong at minimum. An unknown strong line, or perhaps a band-head, at λ 4738 acts similarly. The line is not sharp and is somewhat shaded toward the red. Its width is about 2 A at minimum, and

its wave-length from 45 plates, corrected for the absorption-line velocity of the star, is λ 4738.297. A search among known lines and bands has failed to reveal the origin of this strong absorption line. It seems very probable that it may be the head of an absorption band which falls off toward the red.

## 6. THE RADIAL VELOCITIES DETERMINED FROM ABSORPTION LINES

The measurements for the determination of radial velocity have been shared in part by Mr. Adams and Misses Burwell and Stone, but all the plates have been measured once, and usually twice, by the writer.

TABLE VI

ABSORPTION-LINE NORMAL POINTS

| Weighted Mean Phase | Weighted Mean *V* | Weighted Mean Phase | Weighted Mean *V* |
|---|---|---|---|
| days | km/sec. | days | km/sec. |
| 0..... | +64.0 | 113..... | +57.5 |
| 7..... | 64.3 | 147..... | 54.9 |
| 16..... | 63.7 | 238..... | 53.8 |
| 41..... | 63.0 | 278..... | 59.8 |
| 70..... | 60.9 | 288..... | 61.2 |
| 89..... | +60.0 | 312..... | +63.0 |

Although the dispersion used is less than that desirable for radial velocity measurements involving a comparatively small range, it should be borne in mind that the absorption lines, especially near maximum, are numerous and of superior quality, and that the results are based upon a large number of measures. Furthermore, the absorption lines used throughout the whole period of the star's variation, which are listed in Table V, are unusually homogeneous in character, being without exception strong low-temperature or ultimate lines of a small number of elements. The average number measured on each plate is 12 for the whole series. At minimum fewer lines were available than at maximum. In forming the normal points, weights, based on the number of lines, the number of measures, the dispersion employed, and the quality, were assigned to each plate.

After arranging the dark-line velocities given in Table II in order of phase, eleven normal places, of comparable weight, were formed. These are given in Table VI and are represented by open circles on the curve of Figure 4.

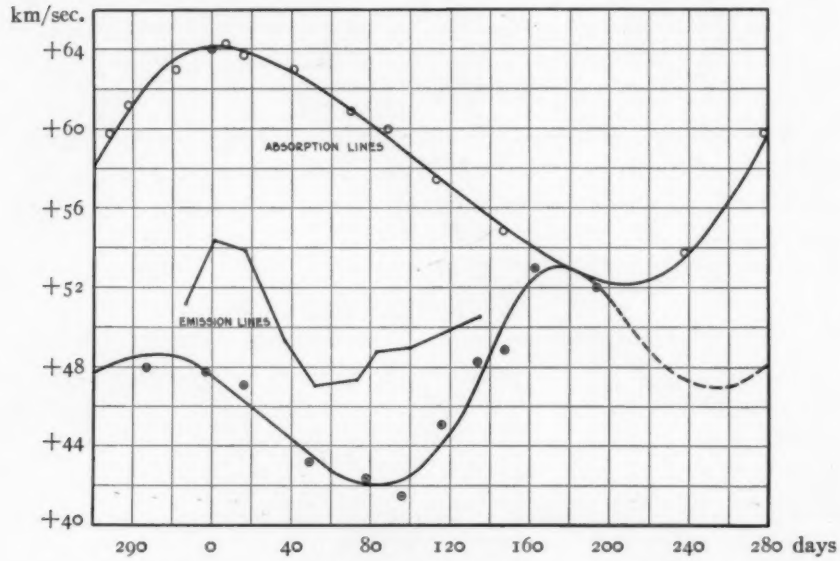



FIG. 4.—Velocity-curves for absorption lines (above) and emission lines (below). The broken middle line represents the velocities of the emission lines of ionized iron.

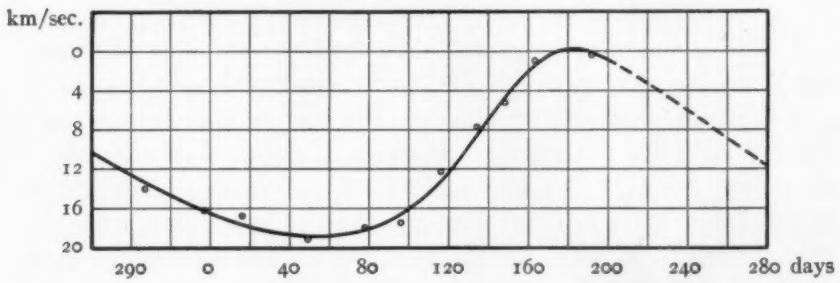



FIG. 5.—Curve showing differences between absorption and emission velocities

Although it is not thought that the variable star is a spectroscopic binary, the curve drawn through the normal points can be described best in terms of the usual elements of an elliptic orbit.

The following elements are satisfactory. A least-squares solution has not been attempted.

$$P = 330 \text{ days}$$
$$K = 5.9 \text{ km/sec.}$$
$$\gamma = +58.2 \text{ km/sec.}$$
$$\omega = 265°.2$$
$$e = 0.20$$
$$a \sin i = 26{,}200{,}000 \text{ km}$$
$$\frac{m_1^3 \sin^3 i}{(m+m_1)^2} = 0.007 \odot$$

The striking feature of the velocity-curve is its resemblance in form to the light-curve. The maximum positive velocity occurs at maximum, and the greatest velocity of approach at minimum, light. The case is thus opposite in phase to that of the Cepheids, for which maximum velocity of approach occurs at maximum of light.

Previous observers[1] have not failed to consider the possibility of variable radial velocity but have made observations only at the star's brightest phases, which correspond to the broad maximum of the velocity curve. The evidence has accordingly pointed to the conclusion that the velocity was constant.

Thus Campbell's observations[2] in 1897 and 1898, including 7 plates with phases from $-36$ to $+55$ days from maximum, show a range of only 3 km/sec. His results, which were obtained with three prisms, are given in Table VII. In themselves, these observations show no indication of variation in velocity, but their deviations from the Mount Wilson curve are not larger than might be attributed to error in observation. On the other hand, the series of single-prism observations by Stebbins[3] in 1902, with phases up to $+83$ days, show a distinct tendency, when reduced to velocities, to follow the variable velocity-curve here given.

Merrill has made numerous observations of the velocities of other long-period variables, but as these photographs likewise

[1] Küstner, *Astrophysical Journal*, **27**, 301, 1908; Plaskett, *Journal of the Royal Astronomical Society of Canada*, **1**, 45, 1907; Merrill, *Publications of the Detroit Observatory*, **2**, 65, 1916; Lunt, *Astrophysical Journal*, **48**, 265, 1918; Harper, *Publications of the Dominion Observatory*, **4**, 334, 1920; Frost and Lowater, *Astrophysical Journal*, **58**, 265, 1923.

[2] *Astrophysical Journal*, **9**, 31, 1899. [3] *Lick Observatory Bulletins*, **2**, 81, 1903.

show the absorption spectrum only near maximum, there is no evidence at hand from other stars of this class. He has reached the conclusion that in the case of X Ophiuchi,[1] where a companion star checks the velocity of the variable, and probably in other stars of this class, the absorption lines give the true velocity of the star. The present results for *o* Ceti, if typical, complicate the problem, inasmuch as it raises the question whether the velocity of translation of the star is the $\gamma$-velocity of the curve or the velocity at either maximum or minimum. Statistical methods may ultimately furnish the answer; but it is impossible at present, on account of the small range

TABLE VII

OBSERVATIONS BY CAMPBELL

| | Phase | *V* | C.−Mt. W. |
|---|---|---|---|
| Nov. 10—1897..... | −20 days | +63.4 km/sec. | +0.2 |
| Dec. 15..... | +15 | 62.0 | −1.8 |
| 15..... | +15 | 60.7 | −3.1 |
| Aug. 29—1898..... | −36 | 62.8 | +1.1 |
| Sept. 4..... | −30 | 63.7 | +1.3 |
| 19..... | −15 | 61.8 | −1.7 |
| Nov. 29..... | +55 | 62.3 | +0.4 |

of variation and the lack of direct methods of attack, to come to any conclusion.

The variable X Ophiuchi is visible for so short a period that it is not possible to determine whether the absorption-line velocity varies or not. The companion of Mira is so faint and so difficult to separate spectrographically that it cannot be satisfactorily used for determining the velocity of the system. The only evidence available is that from the bright lines of the companion, shown on a few plates mostly of small dispersion, which give the same velocity as the variable (both bright and dark lines) at minimum. It is possible that the bright lines are displaced to the violet, as in B.D. +11°4673. Merrill's[2] computations would seem to indicate that for the Me stars in general a small value of the K-term is given by use of the velocities of the absorption lines at maximum.

[1] *Mt. Wilson Contr.*, No. 261; *Astrophysical Journal*, **57**, 251, 1923.

[2] *Mt. Wilson Contr.*, No. 264; *Astrophysical Journal*, **58**, 255, 1923.

## 7. EMISSION LINES

The emission lines of hydrogen in *o* Ceti were first noticed[1] in 1887 by Pickering as one of the earliest results of the photographic observation of stellar spectra with the objective prism at the Harvard College Observatory. Later, other long-period variables were found to have the same bright lines. This characteristic then became the criterion by which numerous variables of this type were detected from inspection of their spectra, and thus far only a few M-type stars emitting bright hydrogen lines have failed to exhibit the Mira type of variation in light. The exceptional cases include six of the faintest dwarfs: W.B. $10^h234$, $16^h906$, Furujhelm 54 and 58, and the companions of Castor[2] and *o* Eridani;[3] and six giants:[4] W Cephei, Boss 1985, 5650, H.D. 42474, B.D. $+61°8$, and C.D.M. $-33°16843$. The dwarfs are not known to vary in brightness. Some of the giants have an irregular variation of small range. All but the last star have spectra which are essentially different from that of *o* Ceti and need not concern us in this discussion. C.D.M. $-33°16843$, to which our attention was directed by Miss Cannon, gives, however, a typical Me spectrum, yet among several hundred stars of this type it is the only one known to maintain constant brightness.

The hydrogen lines on our plates are usually considerably overexposed in order that the absorption lines of the spectrum may be measured. The Balmer series has been recorded from $H\alpha$ to $H\iota$ but the region for which this study is fairly complete extends only from $\lambda$ 3950 to $\lambda$ 5000. The behavior varies with the line and also depends to some extent upon the particular cycle considered, but, in general, the magnitude of the star seems to be the controlling factor. The cycles repeat themselves with only slight inconsistencies.

When the star reaches a minimum none of the hydrogen lines is bright. With increase in light there is a sudden appearance of hydro-

[1] *First Annual Report of the Photographic Study of Stellar Spectra*, Cambridge, 1887; reprinted in *Nature*, **36**, 32, 1887.

[2] Adams and Joy, *Publications of the Astronomical Society of the Pacific*, **32**, 158, 1920; **34**, 174, 1922.

[3] Leonard, *ibid.*, **33**, 272, 1921.

[4] Adams and Joy, *ibid.*, **33**, 263, 1921; Adams, Joy, and Humason, *ibid.*, **34**, 175, 1922; **37**, 161, 1925.

gen emission at about the seventh magnitude, which is first seen in $H\delta$ and, a short time later, in $H\gamma$. In the course of a week or two, these two lines attain great strength, $H\delta$ remaining the stronger until well past maximum. The greatest intensity is reached about a month after maximum light. The decline then sets in; $H\gamma$ becomes somewhat stronger than $H\delta$, and both disappear together near magnitude 8.0. Near minimum the emission lines go over into absorption lines. The relative changes in the hydrogen lines are illustrated in Plate XIX.

$H\alpha$ is present as a bright line on the few plates sensitized for the red portion of the spectrum. Relative to the continuous spectrum, it appears to be weaker than $H\beta$. Shane[1] found $H\alpha$ nearly equal to $H\beta$ in intensity. He also noted that $H\alpha$ was displaced about 1 A toward the violet, relative to the other bright lines, on all his plates. This displacement appears to be confirmed, in part at least, by rough measures of the Mount Wilson plates.

$H\beta$ and $H\zeta$ have been seen and measured each cycle. They make their appearance near the time of maximum, and, on account of their weakness, vanish after three or four months, at about magnitude 6.5. $H\zeta$ seems to reach its maximum somewhat later than $H\beta$. $H\epsilon$ and $H\eta$ are similar in strength on our plates. They appear about a month after maximum and fade out at about the same time as $H\beta$ and $H\zeta$. $H\theta$ and $H\iota$ were found on two plates only, at phases of 49 and 77 days.

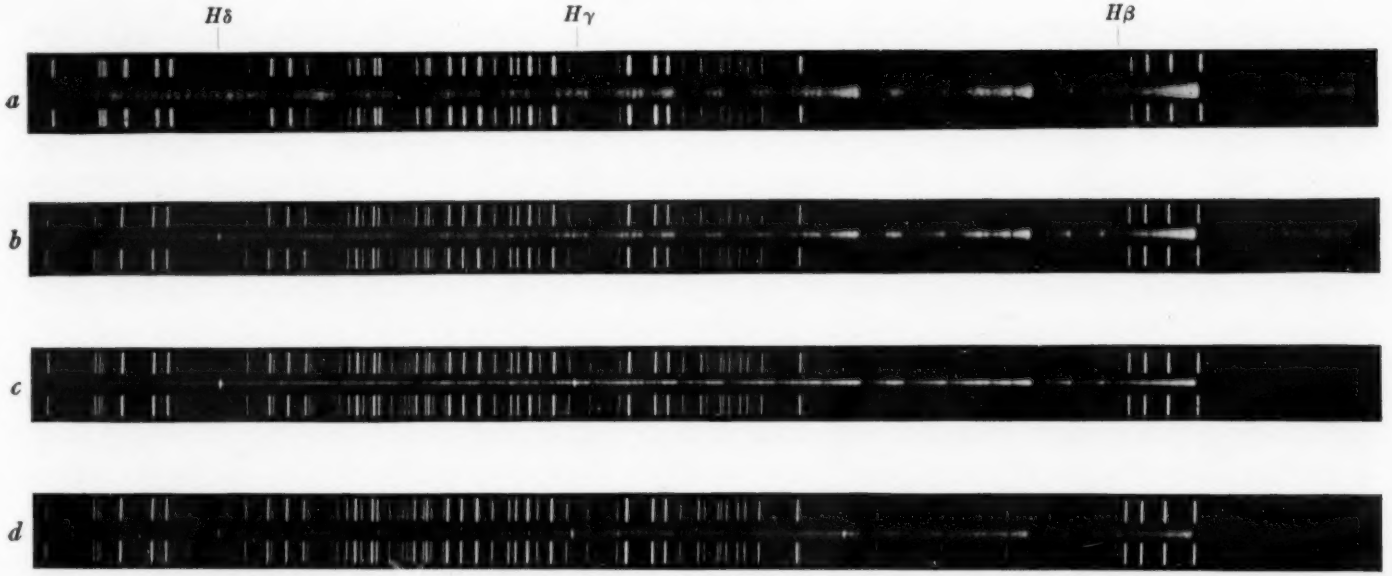
In a general way the hydrogen series has its greatest strength at $H\delta$ and declines toward red and violet. The weakness of $H\epsilon$, $H\kappa$, $H\lambda$, $H\mu$, and $H\xi$ are explained by local absorption, as suggested by Miss Clerke for $H\epsilon$, and by Shane for the other lines.

The hydrogen lines are fairly sharp on one-prism spectrograms and there is little indication of any lack of symmetry. Higher dispersion reveals $H\gamma$ and $H\delta$ to be composite, each consisting of three components. The separation of these components seems to be constant, but the relative intensity has changed since they were first observed. Campbell[2] found at the maxima of 1898 that $H\gamma$ was made up of a strong central component with a lesser one to the violet

[1] *Lick Observatory Bulletins*, **10**, 131, 1922; *Publications of the Astronomical Society of the Pacific*, **32**, 234, 1920.

[2] *Astrophysical Journal*, **9**, 31, 1899.

## PLATE XIX




SPECTROGRAMS OF MIRA ILLUSTRATING THE BEHAVIOR OF THE HYDROGEN EMISSION LINES DURING A CYCLE

(*a*) C 3511, September 8, 1925; magnitude 8.8, phase 248 days. No hydrogen emission lines are present except *Hβ*, which emanates from the companion.

(*b*) γ 6120, August 27, 1917; magnitude 5.9, phase 284 days. *Hδ* stronger than *Hγ*.

(*c*) γ 6306, October 26, 1917; magnitude 3.7, phase 14 days.

(*d*) γ 6724, March 2, 1918; magnitude 7.1, phase 141 days. *Hγ* stronger than *Hδ*.

and a weak one on the red side. *Hδ* was symmetrical with a strong central line and two equal side components. Wright[1] was able to see but two components of *Hγ* in 1899, but when he examined it in 1909 for indications of the Zeeman effect it was triple, with much the same intensity relations as found by Campbell in 1898. On plates taken with three prisms by Mr. Merrill and the writer at the last maximum in December, 1925, the three components of *Hγ* are not well resolved, but seem to decrease slightly in intensity toward the red, the violet component being the strongest. *Hδ*, however, shows a strong central line with a very weak companion to the violet and a line of intermediate intensity well separated on the red side. Plate XX*f* shows its appearance on November 25, 1925. In the succeeding three months the central component of *Hδ* strengthened relatively, so that the close violet component was almost lost. There was no certain change in *Hγ*. The separations are practically the same as those found by Campbell, but the intensities are quite different. The whole line or band shifts its position in accordance with the velocity-curve given in Section 8. As far as observed, the other bright lines have been found single.

No explanation can be suggested which satisfactorily accounts for the peculiar structure of these hydrogen lines. The components are unsymmetrical in position and intensity. The separations of the components are 0.29 A to the violet side and 0.46 A to the red. The Zeeman and Stark effects seem to be ruled out. Since they shift as a unit, the components must originate in the same portion of the star's atmosphere. Unsymmetrical double reversal is a possible cause.

In addition to the emission lines of hydrogen, numerous other bright lines have been observed. Campbell[2] in 1897 and 1898 first measured λλ 4308 and 4376 and identified them, from 3-prism plates, with the corresponding iron lines. He concluded that they have the same displacements as the bright hydrogen lines. He also called attention to the bright line λ 4103 which Plaskett[3] later identified with Rowland's line λ 4103.1, *Si Mn*. Plaskett refers to seven other

[1] *Lick Observatory Bulletins*, **6**, 60, 1910.

[2] *Astrophysical Journal*, **9**, 31, 1899.

[3] *Journal of the Royal Astronomical Society of Canada*, **1**, 56, 1907.

bright lines in addition to the hydrogen lines. Numerous emission lines are recorded by Stebbins, Küstner, Adams and Joy, and Frost and Lowater. Merrill finds that most of these lines occur in other long-period variable stars.

All the bright lines and bright places resembling lines, which have been measured, are listed in Table VIII. The wave-lengths from the Mount Wilson measures, given in the first two columns, have been corrected by the mean displacement of any or all of the following bright lines measured on the plate: $H\delta$, $H\gamma$, λλ 4202, 4216, 4291, 4308, 4376, 4571. These are among the strongest and most frequently recurring bright lines, and there can be no question of the correctness of their identification[1] with hydrogen and the low-temperature radiation of iron and magnesium.

The validity of the assumption that all the bright lines are subject to the same displacement is still open to question, but the agreement found here for the hydrogen lines and the low-temperature lines makes it reasonably certain that this is the case for these groups at least. The case is not so clear for some of the remaining lines, such as λ 4233 of ionized iron and λλ 3905 and 4103 of silicon, which belong to higher temperature classes and probably originate at different levels. For the many lines with no plausible identifications the corrections are presumably in the right direction.

The third column of Table VIII gives the probable error of the mean wave-lengths computed from the internal agreement of the measures, the fourth column the probable identification, and the fifth, the number of Mount Wilson plates upon which the wave-length is based. The line may have been present on other plates as well, but omitted in measurement on account of its weakness or poor definition. No lines are included which have not been found on at least three plates, unless measured by other observers.

The remaining columns contain the wave-lengths published by other observers. In the last column are the wave-lengths of the bright lines found by Merrill[2] in long-period variables other than *o* Ceti. The values which are starred have been supplied by Mr.

[1] Adams and Joy, *Publications of the Astronomical Society of the Pacific*, **35**, 168, 1923.

[2] *Mt. Wilson Contr.*, No. 265; *Astrophysical Journal*, **58**, 197, 1923.

## TABLE VIII

EMISSION LINES IN *o* CETI

| WAVE-LENGTH | | PROBABLE ERROR | IDENTIFICATION | NO. PLATES | STEBBINS | PLASKETT | FROST AND LOWATER | ME STARS MERRILL |
|---|---|---|---|---|---|---|---|---|
| Rowland | I.A. | | | | | | | |
| ..... | ..... | ..... | *Hκ* | ..... | 3750.5 | ..... | ..... | ..... |
| 3770.83 | .70 | ±0.04 | *Hι* | 2 | 3770.85 | ..... | ..... | ..... |
| 3797.96 | .83 | .00 | *Hθ* | 2 | 3798.09 | ..... | ..... | ..... |
| 3835.65 | .52 | .03 | *Hη* | 16 | 3835.53 | ..... | ..... | Standard |
| 3852.90 | .77 | .07 | ..... | 2 | 3852.84 | ..... | ..... | 3852.76 |
| 3889.33 | .20 | .01 | *Hζ* | 31 | 3889.24 | ..... | Standard | Standard |
| 3905.82 | .69 | .02 | *Si* | 13 | 3905.69 | ..... | 3905.78 | Standard |
| 3907.83 | .70 | .02 | ..... | 2 | 3907.51 | ..... | ..... | 3907.60* |
| ..... | ..... | ..... | ..... | ..... | 3932.78 | ..... | ..... | ..... |
| 3938.57 | .44 | .04 | ..... | 4 | 3938.43 | ..... | ..... | 3938.47* |
| ..... | ..... | ..... | ..... | ..... | 3967.82 | ..... | ..... | 3967.66* |
| 3970.30 | .16 | .03 | *Hε* | 11 | ..... | ..... | Standard | Standard |
| ..... | ..... | ..... | ..... | ..... | 3978.0 | ..... | ..... | 3977.92 |
| ..... | ..... | ..... | ..... | ..... | 3997.8 | ..... | ..... | 3997.87* |
| ..... | ..... | ..... | ..... | ..... | 4002.0 | ..... | ..... | 4001.90* |
| 4006.94 | .80 | .03 | ..... | 9 | 4007.07 | ..... | ..... | ..... |
| 4030.75 | .61 | .07 | *Mn* | 2 | 4030.8 | ..... | ..... | 4030.66 |
| 4101.89 | .74 | .00 | *Hδ* | 51 | 4101.99 | Standard | Standard | Standard |
| 4103.15 | .00 | .01 | *Si* | 39 | ..... | 4103.20 | 4103.15 | 4103.18 |
| 4119.75 | .60 | .02 | ..... | 10 | ..... | 4119.81 | ..... | 4119.65* |
| 4122.92 | .77 | .03 | ..... | 6 | ..... | ..... | ..... | 4122.98* |
| 4138.75 | .60 | .02 | *Fe*+ | 23 | ..... | 4138.78 | 4138.80 | 4138.80 |
| 4166.00 | .84 | .01 | ..... | 16 | ..... | 4166.09 | 4166.03 | 4166.09* |
| 4170.81 | .65 | .05 | ..... | 5 | ..... | ..... | ..... | 4171.04* |
| 4173.63 | .47 | .04 | *Fe*+ | 3 | ..... | 4173.83 | ..... | 4173.60* |
| 4178.98 | .82 | .05 | *Fe*+ | 35 | 4178.9 | 4179.09 | ..... | 4178.98 |
| 4202.18 | .02 | .01 | *Fe* | 32 | 4202.24 | ..... | 4202.01 | Standard |
| 4206.86 | .70 | .03 | *Fe* | 10 | ..... | ..... | 4206.68 | ..... |
| 4216.35 | .19 | .01 | *Fe* | 18 | 4216.04 | ..... | ..... | 4215.90 |
| 4229.47 | .31 | .02 | ..... | 4 | ..... | 4229.76 | ..... | ..... |
| 4233.49 | .33 | .01 | *Fe*+ | 40 | 4233.45 | 4233.61 | 4233.42 | 4233.51 |
| 4249.49 | .34 | .06 | ..... | ..... | ..... | ..... | ..... | ..... |
| 4258.45 | .30 | .04 | *Fe* | 7 | ..... | ..... | ..... | ..... |
| 4291.64 | .49 | .01 | *Fe* | 10 | ..... | ..... | ..... | ..... |
| 4308.10 | .94 | .01 | *Fe* | 30 | 4308.03 | ..... | ..... | Standard |
| 4340.64 | .48 | .00 | *Hγ* | 58 | 4340.66 | Standard | Standard | Standard |
| 4352.52 | .36 | .04 | ..... | 9 | ..... | ..... | ..... | 4352.61* |
| 4372.74 | .58 | .03 | ..... | 12 | 4372.94 | ..... | ..... | 4372.74 |
| 4376.09 | .93 | .01 | *Fe* | 31 | 4376.11 | 4375.98 | ..... | 4376.00 |
| 4427.49 | .33 | .02 | *Fe* | 11 | 4427.2 | ..... | ..... | ..... |
| 4434.16 | .00 | .03 | ..... | 3 | 4433.8 | ..... | ..... | ..... |
| ..... | ..... | ..... | ..... | ..... | ..... | ..... | 4454.41 | ..... |
| ..... | ..... | ..... | ..... | ..... | ..... | ..... | 4457.06 | ..... |
| 4458.95 | .79 | .03 | ..... | 20 | ..... | ..... | 4458.79 | 4458.99 |
| 4461.47 | .31 | .03 | ..... | 12 | ..... | ..... | 4461.44 | 4461.52* |
| 4461.75 | .59 | .02 | *Fe* | 9 | ..... | ..... | ..... | ..... |
| 4482.27 | .11 | .10 | *Fe* | 5 | ..... | ..... | ..... | ..... |
| 4489.73 | .57 | .06 | *Fe* | 4 | ..... | ..... | ..... | ..... |
| 4511.66 | .50 | .02 | *In* | 13 | 4511.9 | ..... | 4511.73 | 4511.62 |
| 4521.56 | .40 | .02 | ..... | 4 | ..... | ..... | 4521.54 | 4521.70 |
| 4533.95 | .78 | ±0.09 | ..... | 4 | ..... | ..... | ..... | ..... |

TABLE VIII—*Continued*

| Rowland | I.A. | Probable Error | Identification | No. Plates | Stebbins | Plaskett | Frost and Lowater | Me Stars Merrill |
|---|---|---|---|---|---|---|---|---|
| ..... | ..... | ..... | ..... | ..... | ..... | ..... | 4559.88 | ..... |
| ..... | ..... | ..... | ..... | ..... | ..... | ..... | 4562.15 | ..... |
| 4571.31. | .14 | ±0.01 | *Mg* | 27 | 4571.15 | ..... | ..... | Standard |
| 4578.96. | .79 | .05 | ..... | 3 | ..... | ..... | ..... | 4579.01 |
| 4584.05. | .88 | .20 | *Fe*+ | 2 | ..... | ..... | 4584.08 | ..... |
| ..... | ..... | ..... | ..... | ..... | ..... | ..... | 4633.72 | ..... |
| ..... | ..... | ..... | ..... | ..... | ..... | ..... | 4634.94 | ..... |
| ..... | ..... | ..... | ..... | ..... | ..... | ..... | 4639.26 | ..... |
| ..... | ..... | ..... | ..... | ..... | ..... | ..... | 4756.72 | ..... |
| ..... | ..... | ..... | ..... | ..... | ..... | ..... | 4801.33 | ..... |
| ..... | ..... | ..... | ..... | ..... | ..... | ..... | 4803.18 | ..... |
| ..... | ..... | ..... | ..... | ..... | ..... | ..... | 4838.68 | ..... |
| 4861.49. | .31 | .02 | *Hβ* | 26 | 4861.67 | Standard | Standard | Standard |
| 4923.95. | .78 | .07 | *Fe*+ | 6 | ..... | ..... | ..... | ..... |
| 5018.62. | .45 | ±0.23 | *Fe*+ | 4 | ..... | ..... | ..... | ..... |

Merrill from manuscript. In many cases the lines were measured on three or four plates only, but it is of great interest to note that they are found in other stars.

Wave-lengths are given on the Rowland system, except in the second column where the International values are given for comparison. Stebbins' wave-lengths are corrected by −0.67 A to allow for the displacement of the bright lines, while a correction of +0.25 A suggested by Plaskett has been applied to his wave-lengths to reduce them to the bright hydrogen lines as standards. Merrill omits the wave-lengths of six hydrogen lines and of λλ 3905, 4202, 4308, and 4571, used as standards by which the displacements of the other lines are corrected.

In some cases it is not easy to determine whether a measured point is actually an emission line or a narrow region of continuous spectrum between two absorption lines; but if the line has been seen by different observers under different instrumental conditions and with different dispersions there can be little doubt that the emission is real.

Of the 66 lines given in Table VIII, 49 have been measured on our plates and, after careful examination, have been considered to be real emission lines. The 17 remaining lines recorded by other observers, excluding *Hκ*, which is too far in the violet, have been looked

for on our plates, but either they are not present or else appear to be continuous background rather than real lines. Nine lines are included which are not recorded by other observers; of these 7 have been identified with reasonable certainty.

It is of interest to consider the strength of the bright lines and the time of their appearance with respect to changes in brightness of the star. Table IX gives the relative intensities estimated on an arbitrary scale, as found on well-exposed plates chosen as representative of the line-intensities at different magnitudes and phases. The

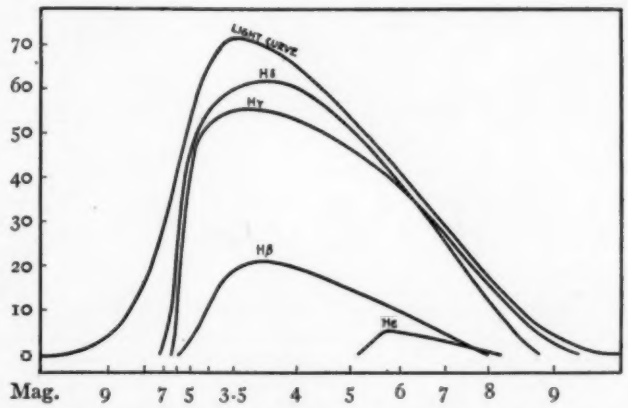



FIG. 6.—Intensities of hydrogen lines for different magnitudes. The ordinates are relative intensities on an arbitrary scale. The scale of the abscissae is given in magnitudes projected from the light-curve.

estimates must be regarded as rough comparisons only, since the atmospheric transmission, as well as photographic and instrumental effects, has not been taken into account. On the whole, it has seemed better to arrange the plates in the order of magnitude rather than phase, since temperature in the star is probably the controlling influence in the variation of the emission. The intensities observed follow the magnitudes more consistently than any other single factor.

The strength of the continuous spectrum on a scale of 5 is given in the second column. The following columns give data for the 36 strongest and most important lines. Some of these appear as absorption lines for a portion of the cycle. The intensities of absorption are then preceded by the letter *a*. Asterisks in the table indicate that the

line is not to be expected on account of over- or underexposure at that point in the spectrum.

Figures 6 and 7 represent graphically the changes in intensity of several of the lines. In order that the intensities may be compared with the normal light variation of the star, the mean light-curve has been included in Figure 6, and the scale of the abscissae is given in magnitudes projected down from the light-curve. Thus, intensities are plotted according to magnitude, but the horizontal spacing is the time scale of the light variation.

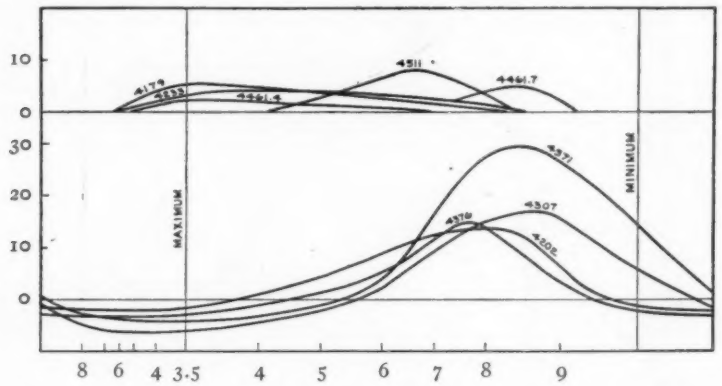



FIG. 7.—Intensities of emission lines for different magnitudes. The scale and coordinates are the same as in Fig. 6. In the lower part of the figure the curves below the zero-line represent the intensity of absorption on an independent scale.

Inspection of Table IX shows two principal groups of bright lines. The first, whose lines are strongest near the maximum of the star, includes the hydrogen series and the enhanced iron lines; the second comprises the low-temperature lines which have been attributed to iron, magnesium, manganese, and indium, and have their maximum intensity four to six months after maximum light.

*Hε* is apparently an exceptional case in the hydrogen series for it reaches its greatest intensity very much later than the other hydrogen lines. This peculiarity may be accounted for on the basis of Miss Clerke's[1] explanation of its weakness. The H line of calcium is so strong at the maximum of the star that it absorbs the radiation

[1] *Problems in Astrophysics*, p. 226, 1903.

## TABLE IX

INTENSITIES OF EMISSION LINES

| Plate | Int. | Mag. | Phase | $H_7$ | $H\zeta$ | 3905 | $H\epsilon$ | 4006 | 4030 | $H\delta$ | 4103 | 4119 | 4122 | 4138 | 4166 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C 3068 | 3 | 8.4 | 272 | * | * | * | .... | .... | a5 | .... | .... | .... | .... | .... | .... |
| C 2013 | 4 | 8.3 | 230 | * | * | * | .... | .... | a8 | .... | .... | .... | .... | .... | .... |
| C 2040 | 3 | 7.8 | 258 | * | * | * | .... | .... | a8 | 1 | .... | .... | .... | .... | .... |
| C 1597 | 2 | 7.1 | 289 | * | * | * | * | * | a8 | 2 | .... | .... | .... | .... | .... |
| 12363 | 4 | 5.9 | 303 | * | * | * | .... | .... | a8 | 8 | 2 | .... | .... | 1 | 1 |
| 11564 | 4 | 5.5 | 281 | * | 1 | * | .... | 1 | a8 | 35 | 2 | .... | 1 | 2 | 1 |
| C 2090 | 5 | 5.0 | 283 | * | 1 | * | .... | 2 | a8 | 70 | 2 | .... | 2 | 5 | 2 |
| 12447 | 4 | 4.9 | 317 | * | * | * | * | 1 | a7 | 25 | 2 | 2 | 2 | 5 | 4 |
| 11609 | 5 | 3.8 | 294 | * | * | * | .... | .... | a8 | 60 | 2 | 1 | 2 | 3 | 3 |
| 5258 | 3 | 3.7 | 326 | * | 2 | * | .... | .... | a8 | 50 | 3 | 2 | 1 | 4 | 1 |
| 6235 | 4 | 3.6 | 318 | * | 1 | * | .... | .... | a8 | 40 | 2 | 1 | 2 | 5 | 2 |
| 11622 | 4 | 2.9 | 311 | * | * | * | * | * | a4 | 55 | 2 | 2 | 3 | 10 | 4 |
| C 500 | 2 | 3.1 | 1 | * | * | * | * | * | * | 60 | 1 | .... | .... | 2 | 3 |
| 9395 | 5 | 3.4 | 29 | 1 | 15 | 1 | .... | 5 | a8 | 90 | 1 | 1 | 2 | 6 | 3 |
| 7413 | 3 | 3.5 | 16 | * | 4 | * | .... | 1 | a6 | 35 | 2 | 2 | 2 | 5 | 3 |
| 6306 | 3 | 3.7 | 14 | 1 | 4 | 1 | .... | 3 | a7 | 35 | 2 | 1 | 1 | 5 | 3 |
| 5290 | 3 | 3.8 | 19 | 2 | 10 | 1 | .... | 5 | a6 | 55 | 2 | 1 | 1 | 6 | 3 |
| 13201 | 4 | 3.8 | 8 | * | 2 | * | * | * | * | 80 | 3 | 1 | 1 | 3 | 2 |
| 6369 | 4 | 4.1 | 42 | 1 | 15 | * | .... | 4 | a7 | 70 | 3 | 1 | 2 | 5 | 2 |
| C 2663 | 2 | 4.7 | 3 | * | 1 | * | * | * | a7 | 45 | 3 | 1 | 1 | 2 | 2 |
| 10276 | 2 | 4.9 | 51 | * | 3 | * | * | * | * | 30 | 1 | 1 | 1 | 2 | 1 |
| 6531 | 4 | 5.1 | 82 | 1 | 6 | * | .... | * | a2 | 40 | 4 | .... | 1 | 1 | .... |
| 7551 | 3 | 5.2 | 70 | 1 | 10 | 1 | .... | 1 | a3 | 70 | 3 | .... | 1 | .... | .... |
| 8789 | 3 | 5.3 | 72 | 2 | 10 | 1 | 1 | 1 | a2 | 35 | 3 | .... | .... | 1 | .... |
| 5559 | 2 | 5.5 | 85 | * | * | * | * | * | * | 25 | 2 | .... | .... | .... | .... |
| 6594 | 4 | 5.7 | 103 | 3 | 18 | 2 | 5 | .... | a2 | 35 | 2 | .... | .... | 1 | .... |
| 9567 | 3 | 6.0 | 86 | 1 | 12 | 1 | 1 | .... | a2 | 40 | 3 | .... | .... | 1 | .... |
| 8805 | 4 | 6.2 | 92 | 1 | 10 | 2 | 3 | .... | 1 | 30 | 3 | .... | .... | 1 | .... |
| 10381 | 3 | 6.4 | 86 | 1 | 10 | 1 | 2 | .... | .... | 35 | 3 | .... | .... | 1 | .... |
| 8872 | 1 | 6.6 | 102 | * | * | * | * | * | * | 12 | 1 | * | * | * | * |
| 11254 | 3 | 6.8 | 114 | 1 | 10 | 1 | 1 | .... | 1 | 30 | 3 | .... | .... | .... | .... |
| 6724 | 1 | 7.1 | 141 | * | * | * | * | * | * | 12 | * | * | * | * | * |
| 10435 | 1 | 7.2 | 109 | 1 | 4 | 1 | 1 | * | * | 30 | 3 | .... | .... | .... | .... |
| C 1845 | 3 | 7.5 | 134 | * | 1 | * | 2 | * | * | 35 | 3 | .... | .... | .... | .... |
| 10503 | 3 | 7.6 | 120 | 2 | 6 | 5 | 4 | .... | 1 | 30 | 5 | .... | .... | .... | .... |
| 7795 | 2 | 7.8 | 133 | * | * | * | 2 | * | * | 15 | 3 | .... | .... | .... | .... |
| C 760 | 1 | 8.1 | 145 | * | * | * | * | * | * | 2 | 1 | * | * | * | * |
| C 1897 | 1 | 8.2 | 162 | * | * | * | * | * | * | 2 | .... | .... | .... | .... | .... |
| C 801 | 1 | 8.4 | 154 | * | * | * | * | * | * | * | * | * | * | * | * |
| C 246 | 1 | 8.5 | 164 | * | * | * | * | * | * | * | * | * | * | * | * |
| C 250 | 1 | 8.6 | 165 | * | * | * | * | * | * | * | * | * | * | * | * |
| C 1442 | 2 | 8.8 | 167 | * | * | * | * | * | * | 1 | .... | .... | .... | .... | .... |
| C 1474 | 1 | 9.0 | 182 | * | * | * | * | * | * | * | * | * | * | * | * |
| C 1478 | 1 | 9.2 | 197 | * | * | * | * | * | * | * | * | * | * | * | * |
| C 1488 | 1 | 9.3 | 201 | * | * | * | * | * | * | * | * | * | * | * | * |
| C 867 | 2 | 9.4 | 215 | * | * | * | * | * | * | .... | .... | .... | .... | .... | .... |
| C 878 | 1 | 9.3 | 234 | * | * | * | * | * | * | .... | .... | .... | .... | .... | .... |
| C 3037 | 1 | 8.9 | 243 | * | * | * | * | * | * | .... | .... | .... | .... | .... | .... |

TABLE IX—*Continued*

| Plate | Int. | Mag. | Phase | 4170 | 4173 | 4179 | 4202 | 4206 | 4216 | 4233 | 4258 | 4291 | 4308 | $H\gamma$ | 4352 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C 3068 | 3 | 8.4 | 272 | .... | .... | .... | .... | .... | .... | .... | .... | a1 | a2 | .... | .... |
| C 2013 | 4 | 8.3 | 230 | .... | .... | .... | a1 | .... | .... | .... | a1 | a2 | a3 | .... | .... |
| C 2040 | 3 | 7.8 | 258 | .... | .... | .... | a2 | .... | .... | .... | a1 | a3 | a2 | .... | .... |
| C 1597 | 2 | 7.1 | 289 | .... | .... | .... | a2 | .... | .... | .... | a2 | a2 | a2 | .... | .... |
| 12363 | 4 | 5.9 | 303 | 1 | .... | 1 | a2 | a2 | .... | 2 | a3 | a3 | a5 | 12 | .... |
| 11564 | 4 | 5.5 | 281 | 1 | 1 | 2 | a2 | a2 | .... | .... | a2 | a4 | a6 | 40 | .... |
| C 2090 | 5 | 5.0 | 283 | 2 | .... | 2 | a3 | a2 | .... | .... | a2 | a3 | a4 | 60 | .... |
| 12447 | 4 | 4.9 | 317 | .... | .... | 1 | a2 | a3 | .... | 1 | a2 | a4 | a2 | 35 | .... |
| 11609 | 5 | 3.8 | 294 | 3 | .... | 2 | a2 | a2 | .... | 1 | a2 | a4 | a4 | 75 | .... |
| 5258 | 3 | 3.7 | 326 | 1 | 1 | 2 | a2 | a1 | .... | 1 | a2 | a4 | a4 | 45 | .... |
| 6235 | 4 | 3.6 | 318 | 1 | 1 | 3 | a1 | a2 | .... | 2 | a2 | a4 | a2 | 35 | 1 |
| 11622 | 4 | 2.9 | 311 | 3 | 1 | 8 | a2 | a2 | .... | 1 | a2 | a4 | a2 | 80 | .... |
| C 500 | 2 | 3.1 | 1 | 1 | .... | 1 | a2 | a2 | .... | 1 | a2 | a5 | a2 | 55 | .... |
| 9395 | 5 | 3.4 | 29 | 2 | 1 | 5 | .... | a2 | .... | 3 | a2 | a4 | a1 | 80 | .... |
| 7413 | 3 | 3.5 | 16 | 1 | 1 | 4 | .... | a1 | .... | 1 | a1 | a3 | a3 | 30 | .... |
| 6306 | 3 | 3.7 | 14 | 1 | 1 | 5 | .... | a1 | .... | 2 | a1 | a3 | a2 | 25 | 1 |
| 5299 | 3 | 3.8 | 19 | 2 | 1 | 5 | .... | a1 | .... | 2 | a1 | a3 | a2 | 35 | .... |
| 13201 | 4 | 3.8 | 8 | 2 | 1 | 2 | .... | .... | .... | 1 | a3 | a3 | a2 | 60 | .... |
| 6369 | 4 | 4.1 | 42 | 1 | 2 | 5 | 1 | .... | .... | 2 | a1 | a3 | a2 | 35 | 1 |
| C 2663 | 2 | 4.7 | 3 | .... | .... | 1 | 1 | .... | .... | .... | a1 | a3 | a3 | 30 | .... |
| 10276 | 2 | 4.9 | 51 | 1 | 1 | 2 | 1 | .... | .... | 1 | a1 | a2 | a1 | 30 | .... |
| 6531 | 4 | 5.1 | 82 | .... | 1 | 2 | 4 | 1 | 2 | 3 | a1 | a1 | a1 | 50 | .... |
| 7551 | 3 | 5.2 | 70 | 1 | 1 | 3 | 3 | 1 | 1 | 3 | a1 | a2 | .... | 40 | 1 |
| 8789 | 3 | 5.3 | 72 | .... | 1 | 2 | 4 | 1 | 2 | 3 | a1 | a1 | 1 | 40 | .... |
| 5559 | 2 | 5.5 | 85 | .... | .... | 1 | 2 | 1 | 2 | 1 | .... | a2 | .... | 18 | .... |
| 6594 | 4 | 5.7 | 103 | .... | 1 | 2 | 6 | 1 | 2 | 3 | .... | a1 | 2 | 40 | .... |
| 9567 | 3 | 6.0 | 86 | .... | 1 | 2 | 5 | 1 | .... | 3 | .... | a2 | .... | 35 | 1 |
| 8805 | 4 | 6.2 | 92 | .... | 1 | 2 | 15 | 1 | 5 | 4 | 1 | .... | 15 | 40 | .... |
| 10381 | 3 | 6.4 | 86 | .... | 1 | 2 | 5 | .... | 1 | 2 | .... | .... | 1 | 40 | .... |
| 8872 | 1 | 6.6 | 102 | * | * | * | 3 | .... | 1 | 1 | .... | .... | 4 | 20 | .... |
| 11254 | 3 | 6.8 | 114 | .... | 1 | 2 | 5 | 1 | 3 | 1 | .... | 1 | 1 | 30 | .... |
| 6724 | 1 | 7.1 | 141 | * | * | * | 4 | 1 | 2 | 1 | .... | 1 | 6 | 25 | 1 |
| 10435 | 1 | 7.2 | 109 | .... | 1 | 2 | 5 | 1 | 2 | 1 | 1 | 1 | 3 | 20 | .... |
| C 1845 | 3 | 7.5 | 134 | .... | .... | 5 | 18 | 1 | 12 | 2 | 1 | 15 | 10 | 45 | .... |
| 10503 | 3 | 7.6 | 120 | .... | .... | 3 | 12 | 1 | 3 | 3 | 2 | 2 | 12 | 35 | .... |
| 7795 | 2 | 7.8 | 133 | .... | 1 | 2 | 12 | 2 | 5 | 2 | .... | 2 | 5 | 25 | .... |
| C ·760 | 1 | 8.1 | 145 | * | * | * | 5 | * | * | * | 1 | .... | 10 | 5 | .... |
| C 1897 | 1 | 8.2 | 162 | .... | .... | .... | 5 | 3 | 4 | .... | .... | 3 | 8 | 3 | .... |
| C 801 | 1 | 8.4 | 154 | * | * | * | 3 | .... | 1 | .... | .... | 1 | 10 | 1 | .... |
| C 246 | 1 | 8.5 | 164 | * | * | * | 1 | * | * | * | * | * | 3 | .... | .... |
| C 250 | 1 | 8.6 | 165 | * | * | * | 2 | .... | 1 | * | * | 1 | 15 | 2 | .... |
| C 1442 | 2 | 8.8 | 167 | .... | .... | .... | 4 | 1 | 2 | 1 | 1 | 3 | 10 | 4 | .... |
| C 1474 | 1 | 9.0 | 182 | * | * | * | 1 | * | * | * | * | * | 5 | .... | .... |
| C 1478 | 1 | 9.2 | 197 | * | * | * | 1 | * | * | * | * | * | 5 | .... | .... |
| C 1488 | 1 | 9.3 | 201 | * | * | * | * | * | * | * | * | * | 5 | .... | .... |
| C 867 | 2 | 9.4 | 215 | .... | .... | .... | .... | .... | .... | .... | .... | .... | 4 | .... | .... |
| C 878 | 1 | 9.3 | 234 | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... |
| C 3037 | 1 | 8.9 | 243 | .... | .... | .... | .... | .... | .... | .... | .... | .... | 2 | .... | .... |

TABLE IX—*Continued*

| Plate | Int. | Mag. | Phase | 4372 | 4376 | 4427 | 4434 | 4458 | 4461 | 4482 | 4489 | 4511 | 4521 | 4571 | *Hβ* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C 3068 | 3 | 8.4 | 272 | .... | a1 | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... |
| C 2013 | 4 | 8.3 | 230 | .... | a2 | a1 | .... | 2 | 2 | a1 | a2 | .... | .... | .... | .... |
| C 2040 | 3 | 7.8 | 258 | .... | a2 | a1 | .... | 2 | 1 | a2 | a2 | .... | .... | .... | .... |
| C 1597 | 2 | 7.1 | 289 | .... | a2 | .... | .... | 2 | 1 | a2 | a1 | .... | .... | .... | .... |
| 12363 | 4 | 5.9 | 303 | .... | a3 | a1 | .... | .... | .... | a1 | a1 | .... | .... | a2 | 2 |
| 11564 | 4 | 5.5 | 281 | .... | a2 | a2 | 2 | 3 | 2 | a2 | a2 | .... | .... | a4 | 1 |
| C 2090 | 5 | 5.0 | 283 | .... | a2 | a2 | 4 | 2 | 2 | a2 | a2 | .... | .... | a4 | 2 |
| 12447 | 4 | 4.9 | 317 | .... | a2 | a2 | .... | 1 | .... | a1 | a1 | .... | .... | a1 | 1 |
| 11609 | 5 | 3.8 | 294 | .... | a1 | a1 | 4 | 2 | 2 | a2 | a1 | .... | .... | a2 | 20 |
| 5258 | 3 | 3.7 | 326 | .... | a2 | a2 | .... | 1 | 1 | a2 | a2 | .... | .... | a2 | 8 |
| 6235 | 4 | 3.6 | 318 | .... | a2 | a2 | 3 | .... | .... | a2 | a2 | .... | 2 | a4 | 5 |
| 11622 | 4 | 2.9 | 311 | .... | a2 | a2 | 4 | 1 | 1 | a2 | a2 | .... | 4 | a2 | 40 |
| C 500 | 2 | 3.1 | 1 | .... | a2 | a2 | 3 | .... | .... | a3 | a3 | .... | 3 | a4 | 30 |
| 9305 | 5 | 3.4 | 29 | .... | a1 | a2 | 10 | 2 | 2 | a2 | a2 | .... | 10 | a2 | 40 |
| 7413 | 3 | 3.5 | 16 | .... | a2 | a2 | 2 | 1 | 0 | a1 | a2 | .... | .... | a2 | 4 |
| 6306 | 3 | 3.7 | 14 | .... | a2 | a2 | 2 | 4 | 1 | a2 | a3 | 1 | 5 | a2 | 5 |
| 5299 | 3 | 3.8 | 19 | .... | a2 | a2 | 2 | 2 | 3 | a2 | a3 | .... | 15 | a2 | 12 |
| 13201 | 4 | 3.8 | 8 | .... | a1 | a2 | 10 | 2 | 2 | a2 | a2 | .... | 2 | a2 | 10 |
| 6369 | 4 | 4.1 | 42 | .... | a1 | a2 | 10 | 2 | 2 | a1 | a2 | 1 | 3 | a2 | 2 |
| C 2663 | 2 | 4.7 | 3 | .... | a1 | a2 | 1 | 1 | .... | a1 | a2 | 1 | 1 | a1 | 2 |
| 10276 | 2 | 4.9 | 51 | .... | .... | a1 | 1 | .... | .... | a1 | a2 | 1 | .... | .... | 3 |
| 6531 | 4 | 5.1 | 82 | .... | 1 | .... | 1 | 1 | 1 | a1 | a2 | 1 | 2 | .... | 2 |
| 7551 | 3 | 5.2 | 70 | .... | .... | a1 | .... | 2 | 1 | a1 | a1 | 1 | 2 | .... | 2 |
| 8789 | 3 | 5.3 | 72 | .... | 2 | .... | .... | 1 | 2 | a1 | .... | 4 | 1 | 1 | 12 |
| 5559 | 2 | 5.5 | 85 | .... | 1 | .... | .... | 1 | 2 | a1 | a1 | 1 | 1 | 1 | 2 |
| 6594 | 4 | 5.7 | 103 | .... | 4 | 1 | .... | 2 | 2 | a1 | a1 | 2 | .... | 2 | 12 |
| 9567 | 3 | 6.0 | 86 | .... | 2 | 1 | .... | 1 | 1 | .... | .... | 3 | 1 | 1 | 10 |
| 8805 | 4 | 6.2 | 92 | 1 | 10 | 2 | .... | 1 | 2 | .... | 1 | 12 | 1 | 16 | 18 |
| 10381 | 3 | 6.4 | 86 | .... | 2 | 1 | .... | .... | .... | .... | .... | 2 | 1 | .... | 10 |
| 8872 | 1 | 6.6 | 102 | .... | 2 | .... | .... | .... | 1 | .... | .... | 1 | .... | 4 | 3 |
| 11254 | 3 | 6.8 | 114 | .... | 5 | .... | 1 | 2 | 1 | .... | .... | 4 | .... | 2 | 1 |
| 6724 | 1 | 7.1 | 141 | 2 | 10 | 1 | 1 | 1 | 2 | 1 | .... | 1 | .... | 20 | 5 |
| 10435 | 1 | 7.2 | 109 | 1 | 3 | .... | .... | 1 | 1 | .... | .... | 3 | .... | 4 | .... |
| C 1845 | 3 | 7.5 | 134 | 2 | 25 | 2 | 1 | 1 | 2 | .... | .... | 10 | .... | 30 | 1 |
| 10503 | 3 | 7.6 | 120 | 2 | 8 | 1 | 2 | 1 | 1 | 1 | 1 | 4 | .... | 15 | 3 |
| 7795 | 2 | 7.8 | 133 | 4 | 10 | 2 | 1 | 1 | 4 | .... | .... | 4 | .... | 18 | .... |
| C 760 | 1 | 8.1 | 145 | 3 | 4 | 1 | .... | 1 | 2 | .... | .... | 1 | .... | 16 | .... |
| C 1897 | 1 | 8.2 | 162 | 1 | 18 | 2 | .... | 1 | 4 | 1 | 2 | .... | .... | 25 | .... |
| C 801 | 1 | 8.4 | 154 | 1 | 2 | 1 | .... | 1 | 2 | 1 | 2 | .... | .... | 18 | .... |
| C 246 | 1 | 8.5 | 164 | * | 1 | * | * | * | * | * | * | * | * | 15 | .... |
| C 250 | 1 | 8.6 | 165 | .... | 5 | 1 | .... | .... | 2 | 1 | 2 | .... | .... | 30 | .... |
| C 1442 | 2 | 8.8 | 167 | 2 | 5 | 3 | .... | 1 | 4 | 1 | 1 | .... | .... | 25 | .... |
| C 1474 | 1 | 9.0 | 182 | .... | 2 | .... | .... | .... | 1 | .... | .... | .... | .... | 20 | .... |
| C 1478 | 1 | 9.2 | 197 | .... | .... | * | * | * | * | * | * | .... | .... | 20 | .... |
| C 1488 | 1 | 9.3 | 201 | * | * | * | * | * | * | * | * | * | * | 15 | .... |
| C 867 | 2 | 9.4 | 215 | .... | 1 | .... | .... | .... | .... | .... | .... | .... | .... | 18 | .... |
| C 878 | 1 | 9.3 | 234 | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... | 5 | .... |
| C 3037 | 1 | 8.9 | 243 | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... | 10 | .... |

proceeding from $H\epsilon$. As the star decreases in brightness and becomes cooler H diminishes in width and allows the bright $H\epsilon$ to appear. This explanation requires the presence of at least a portion of the calcium gas at levels above the origin of the $H\epsilon$ emission. The change in the strength of H is well illustrated in the reproduction of Stebbins' spectrograms[1] and is likewise shown on several of the plates

[1] *Lick Observatory Bulletins*, **2**, 81, 1903.

used in this investigation. Shane[1] has accounted for the weakness of $H\kappa$, $H\lambda$, $H\mu$, and $H\xi$ by the corresponding absorption of the low-temperature lines of iron and vanadium.

The lines of silicon seem to be less sensitive to temperature changes and do not show a distinct maximum. In the electric furnace King[2] has found that these lines occur at intermediate temperatures corresponding to the iron lines of class III. The lines of the low-temperature group, with the exception of $\lambda$ 4308 and probably $\lambda$ 4202, which are in class II, belong to temperature class I.

The bright lines whose identification seems to be reasonably certain are listed in Table X.

*Hydrogen lines.*—The identification of the hydrogen lines has been accepted since they were first observed by Pickering in 1886. The question of their relative intensities is as yet not completely solved. The Balmer series behaves in stars as a high-temperature series, and it is to be expected that these lines, if they occur at all, will be strengthened by the maximum temperature. As will be seen in Figure 6, the maximum intensity of the hydrogen lines occurs very near maximum light.

*Enhanced lines.*—As has previously been pointed out,[3] enhanced lines are found in giant M-type stars owing chiefly to their low density. They may well occur in the extreme tenuity of the atmosphere of *o* Ceti. Although the agreement of the wave-lengths is not fully convincing, especially in the case of $\lambda$ 4233, the identification with ionized iron may be considered as entirely probable. Several of the enhanced lines found in *o* Ceti are bright in certain S-type stars and in M-type stars such as W Cephei, which are of the same order of temperature as Mira. They also appear together in emission in the bright-line B-type stars. The consideration of the atomic origin of the lines thus identified, as worked out by Russell,[4] has hastened the conclusion. They are without exception the strongest members of the multiplets arising from the lowest levels of the ionized atom

[1] *Lick Observatory Bulletins*, **10**, 133, 1922.

[2] *Publications of the Astronomical Society of the Pacific*, **33**, 106, 1921.

[3] Adams and Joy, *Publications of the Astronomical Society of the Pacific*, **35**, 328, 1923.

[4] Unpublished series relationships in *Fe*+.

and would presumably be the first to be affected by any cause tending to produce emission lines.

TABLE X

IDENTIFICATION OF EMISSION LINES

| MEASURED WAVE-LENGTH ROWLAND | LABORATORY WAVE-LENGTH | | ELEMENT | TEMP. CLASS | ATOMIC ORIGIN | TIME OF MAX. INTENSITY | |
|---|---|---|---|---|---|---|---|
| | Rowland | I.A. | | | | Mag. | Phase |
| 3770.83..... | 3770.75 | .58 | *Hι* | ..... | 2P–11D | 4.4 | 50[d] |
| 3797.96..... | 3798.02 | .85 | *Hθ* | ..... | 2P–10D | 4.4 | 50 |
| 3835.65..... | 3835.51 | .36 | *Hη* | ..... | 2P– 9D | 5.0 | 65 |
| 3889.33..... | 3889.18 | .05 | *Hζ* | ..... | 2P– 8D | 5.0 | 65 |
| 3905.82..... | 3905.66* | .53 | *Si* | III | ..... | 5.2 | 70 |
| 3970.30..... | 3970.21 | .08 | *Hε* | ..... | 2P– 7D | 5.7 | 85 |
| 4030.75..... | 4030.90 | .77 | *Mn* | I | $1s^6$– $1p^6$ | 7.2 | 125 |
| 4101.89..... | 4101.89 | .74 | *Hδ* | ..... | 2P– 6D | 3.8 | 20 |
| 4103.15..... | 4103.10† | .95 | *Si* | III | ..... | 5.2 | 70 |
| 4138.75..... | 4138.58§ | .43 | *Fe*+ | ..... | $2f^4$– $1p^4$ | 3.5 | 0 |
| 4173.63..... | 4173.63 | .47 | *Fe*+ | ..... | $2p^4$– $1d'^4$ | 3.5 | 0 |
| 4178.98..... | 4179.02 | .86 | *Fe*+ | ..... | $2p^4$– $1f'^4$ | 3.6 | 10 |
| 4202.18..... | 4202.19 | .03 | *Fe* | I | $1f^3$– $1g'^3$ | 8.0 | 145 |
| 4206.86..... | 4206.86 | .70 | *Fe* | IA | $1d^5$– $1p'^7$ | 8.0 | 145 |
| 4216.35..... | 4216.35 | .18 | *Fe* | I | $1d^5$– $1p'^7$ | 7.8 | 140 |
| 4233.49..... | 4233.32 | .16 | *Fe*+ | ..... | $2p^4$– $1d'^4$ | 3.8 | 20 |
| 4258.45..... | 4258.48 | .33 | *Fe* | IA | $1d^5$– $1p'^7$ | 7.8 | 140 |
| 4291.64..... | 4291.62 | .47 | *Fe* | IA | $1d^5$– $1p'^7$ | 8.0 | 145 |
| 4308.10..... | 4308.08 | .91 | *Fe* | II | $1f^3$– $1g'^3$ | 8.6 | 165 |
| 4340.64..... | 4340.63 | .47 | *Hγ* | ..... | 2P– 5D | 3.7 | 15 |
| 4376.09..... | 4376.09 | .93 | *Fe* | I | $1d^5$– $1f'^7$ | 7.8 | 140 |
| 4427.49..... | 4427.47 | .31 | *Fe* | I | $1d^5$– $1f'^7$ | 8.0 | 145 |
| 4461.75..... | 4461.82 | .66 | *Fe* | I | $1d^5$– $1f'^7$ | 8.3 | 160 |
| 4482.27..... | 4482.34 | .18 | *Fe* | I | $1d^5$– $1f'^7$ | 8.0 | 145 |
| 4489.73..... | 4489.91 | .75 | *Fe* | IA | $1d^5$– $1f'^7$ | 8.0 | 145 |
| 4511.66..... | 4511.47‡ | .31 | *In* | I | $1p^2$– $1s^2$ | 6.5 | 110 |
| 4571.31..... | 4571.28 | .11 | *Mg* | I | 1S– $1p^3$ | 8.4 | 165 |
| 4584.05..... | 4584.01 | .84 | *Fe*+ | ..... | $2f^4$– $1d'^4$ | 3.5 | 0 |
| 4861.49..... | 4861.51 | .33 | *Hβ* | ..... | 2P– 4D | 3.7 | 15 |
| 4923.95..... | 4924.09 | .92 | *Fe*+ | ..... | $1s^6$– $1p'^6$ | 3.5 | 0 |
| 5018.62..... | 5018.61 | .44 | *Fe*+ | ..... | $1s^6$– $1p'^6$ | 3.5 | 0 |
| ..... | 6563.04 | .82 | *Hα* | ..... | 2P– 3D | ..... | ..... |

* King, unpublished.

† King, *Publications of the Astronomical Society of the Pacific*, **35**, 330, 1923.

§ Calculated from known terms.

‡ *Ibid.*, **37**, 27, 1925.

The strongest and most persistent of the enhanced bright lines are λλ 4233 and 4179. λ 4233 is sharper and better for measurement. As it occurs with great frequency, the measures should be of high accuracy. λ 4179 is stronger than λ 4233 at maximum light but often is not sharp. It seems to be involved in a strong portion of the continuous spectrum, which may affect its observed wave-length.

As shown in Table XI, the enhanced lines of iron, in the mean, are displaced somewhat toward the red as compared with the hydrogen and low-temperature lines used for determining the wave-length correction. In other words, if we assume that the identification of these lines is correct, they have on the average a positive velocity relative to the eight lines used in forming the bright-line curve. As shown in Figure 4, these lines give points falling between the bright- and dark-line curves. Their origin must then be at a level of the star's atmosphere between the low level producing the absorption lines and the higher levels emitting the bright lines of hydrogen and the low-temperature lines.

TABLE XI

WAVE-LENGTHS OF ENHANCED IRON LINES

| Measured Wave-Length | Laboratory Wave-Length | Diff. | Wt. |
|---|---|---|---|
| 4138.75..... | 4138.58 | +0.17 | 2 |
| 4173.63..... | 4173.63 | 0.00 | 1 |
| 4178.98..... | 4179.02 | −0.04 | 2 |
| 4233.49..... | 4233.32 | +0.17 | 4 |
| 4584.08..... | 4584.01 | +0.07 | 1 |
| 4923.97..... | 4924.09 | ..... | ..... |
| 5018.62..... | 5018.61 | ..... | ..... |
| Weighted mean diff. | ..... | +0.09 | ..... |

A similar effect has been pointed out by Merrill[1] for the enhanced iron lines of B.D. +11°4673.

The middle broken line in Figure 4 shows the mean course of the velocities for the enhanced lines λλ 4138, 4173, 4178, and 4233. The lines λλ 4924 and 5018 have a few measurements only and are located in a region of the spectrum where the focus is not good on our plates. They are not satisfactorily determined, as will be seen from their probable errors, and have not been used in forming the mean curve. The individual lines do not agree closely with one another. Hence this curve should not be taken as comparable in accuracy with the other curves, but the evidence points to a solution along these lines.

*Bright lines of silicon and indium.*—Table IX shows that the silicon lines λλ 3905 and 4103, and λ 4511, which may belong to

[1] *Publications of the Astronomical Observatory, University of Michigan*, 2, 74, 1916.

indium, have intensity-curves quite different from those belonging to the low-temperature group. A comparison of these lines, similar to that for the enhanced lines, is given in Table XII. Apparently these lines arise from levels not far from those producing the enhanced iron lines and have corresponding velocities.

The identification of the indium line is still uncertain. This line, together with λ 4101.9, which is coincident with *Hδ*, is one of the ultimate lines of indium. A plate taken by King[1] with the electric furnace shows that these two lines are strong even at a temperature of 1600°. If present in the sun, λ 4511 is very weak; and it has not

TABLE XII

WAVE-LENGTHS OF SILICON AND INDIUM LINES

| Measured Wave-Length | Laboratory Wave-Length | Diff. |
|---|---|---|
| 3905.82 ..... | 3905.66 | +0.16 |
| 4103.15 ..... | 4103.11 | +0.04 |
| 4511.66 ..... | 4511.47 | +0.19 |
| Mean diff..... | ..... | +0.13 |

been found in absorption in Mira at maximum. The atomic weight of indium is high, but like most of the elements which show bright low-temperature lines, it has a low ionization potential. It is unfortunate that the presence of *Hδ* makes it impossible to observe the other member of this pair.

*The low-temperature lines.*—Among the lines which reach their greatest intensity when the star has cooled after its outburst at maximum brightness are several iron lines, λ 4571 of magnesium, and perhaps λ 4030 of manganese. These lines originate at the lowest levels of the atom and maintain a considerable intensity in the electric furnace at very low temperatures.

In view of the interest attaching to the series relationships of the iron multiplets and of the criticism[2] of the identification of these lines, it seems worth while to give the multiplets of iron involved here as worked out by Walters[3] and Delaporte.[4]

[1] *Publications of the Astronomical Society of the Pacific*, **37**, 27, 1925.

[2] Baxandall, *The Observatory*, **46**, 82, 1923.

[3] *Journal of the Optical Society of America*, **8**, 248, 1924.

[4] *Zeitschrift für Physik*, **23**, 135, 1924, and **26**, 1, 1924.

The lines which have been measured as bright in Mira are marked with an asterisk. The arc intensities in the laboratory and King's temperature classification are added. The wave-lengths are I.A.

IRON MULTIPLETS

| | | | | | |
|---|---|---|---|---|---|
| I. | 4232.724 (8) I A | | 4199.990 (9) I A | 4149.77 (calculated) | |
| | | | *4258.322 (10) I A | *4206.703 (12) I A | 4134.433 (1) |
| | | | | *4291.465 (10) I A | *4216.185 (10) I |
| II. | | 4466.557 (12) I | | | |
| *4489.744 (12) I A | | 4471.66 (calc.) | 4435.154 (10) I A | | |
| | | *4482.176 (4) I A | 4445.480 (1) I A | 4389.251 (10) I A | |
| | | | *4461.658 (12) I | 4405.01 (calc.) | 4325.73 (calc.) |
| | | | | *4427.313 (12) I | 4347.239 (1) I A |
| | | | | | *4375.934 (15) I |
| III. | 4325.770 (35) II | | 4250.791 (25) II | 4147.675 (10) III | |
| | | | *4307.910 (35) II | *4202.032 (30) I | |
| | | | | 4271.764 (35) II | |

The four strongest members of multiplet I are bright in Mira. The absence of the diagonal member λ 4232 is puzzling. It may be too faint in the star.

In multiplet II all lines of the diagonal, and no others, appear in emission. λ 4375 is by far the strongest member. λλ 4482 and 4489 are extremely weak. The strong satellites λλ 4435 and 4389 reach the condition where emission and absorption neutralize each other.

Multiplets I and II are inter-system combinations of quintets and septets. Both arise in the d′ level, which is the lowest level known in the neutral iron atom. The inter-system combination seems especially favorable to the production of the bright lines. All these lines belong to King's class I or I A, but they are not among the "raies ultimes" of De Gramont, which are farther in the violet.

Multiplet III is an f–g′ triplet combination arising from the lowest triplet level. Two members only are bright. The absorption of the other members is much diminished at the time when the bright lines have their greatest intensity. λλ 4250.8 and 4271.8 are blended with the neighboring high-temperature iron lines, λλ 4250.1 and 4271.2, respectively, but the latter are relatively weak at the temperature of Mira and should not affect the multiplet lines to any great extent. Merrill[1] has found that at the positive pole of the arc λλ 4202 and 4307 are unchanged in intensity, but that λλ 4250.8,

[1] *Mt. Wilson Contr.*, No. 253; *Astrophysical Journal*, 56, 479, 1922.

4271.8, and 4325[1] are slightly strengthened. This difference is in the direction found in *o* Ceti. The star appears to furnish an exceedingly sensitive test of the response of these lines to different degrees of excitation, and, in turn, their behavior should reveal very accurately the conditions prevailing in the star. Doubtless the most important factor is that of temperature.

With this problem in mind, Mr. King has been kind enough to examine some spectrograms of iron taken with the furnace and carbon plug. He finds that as the temperature of the plug is increased the low-temperature lines are the first to appear in absorption. The line λ 4307 shows easily and λ 4202 very faintly, together with λ 4375 and several of the low-temperature iron lines which are bright in *o* Ceti, with a plug temperature of 2000° and a vapor temperature of about 1600°; but λ 4325 does not appear at all. For some unknown reason λλ 4202 and 4307 are more sensitive to changes in excitation than the other members of the multiplet.[2]

At maximum the lines of this multiplet, as well as the strong low-temperature lines of class I, are the usual type of absorption lines. Lower temperatures following maximum have the effect of increasing the emission, so that the core is filled up until the absorption is entirely neutralized, and later some of the lines appear entirely in emission. These emission lines increase in strength to a maximum at magnitude about 8.0 and then decrease again toward minimum.

The intensities of the strong lines of multiplet III are given in Table XIII and illustrate the behavior of these lines in absorption and emission at different phases of the star's variation. The cases for λλ 4202 and 4307 are somewhat puzzling, but when the behavior of all the other members of the multiplet is considered, the probability for the suggested identification is strengthened.

Several of the lines do not appear actually to become emission lines, but they can no longer be recognized as absorption lines on these plates; of the two emission lines, λ 4202 is the weaker and the first to appear as a bright line; λ 4307 appears later and remains bright until well past minimum.

[1] Unpublished.

[2] Note added to proof: Miss Moore finds that λλ 4202 and 4308 are less winged in the sun than λλ 4325 and 4371. *Astrophysical Journal*, **63**, 6, 1926.

The low-temperature line of magnesium at λ 4571, which is an inter-system combination between a single and threefold level, is strongest at minimum light and remains bright even longer than λ 4307.

*Unidentified lines.*—If we accept the identification of all the 32 lines given in Table X, there remain 18 lines whose origin cannot be traced. Some of these may in reality be only portions of the continuous spectrum giving the appearance of bright lines. Other lines appear to be real, but a search among the laboratory spectra of the elements fails to suggest any likely identification.

TABLE XIII

INTENSITIES IN f–g′ IRON TRIPLET

| Plate | Phase | Mag. | 4147 | 4202 | 4250 | 4271 | 4307 | 4325 |
|---|---|---|---|---|---|---|---|---|
| γ 8597 | 13 | 3.4 | a1 | a1 | a4 | a3 | a1 | a5 |
| γ 5492 | 77 | 5.3 | a1 | e4 | a2 | a3 | o | a5 |
| γ10503 | 120 | 7.6 | a1 | e12 | a1 | a2 | e12 | a2 |
| C 2383 | 156 | 8.4 | o | e10 | o | o | e20 | o |
| C 867 | 215 | 9.4 | o | o | a1 | a1 | e4 | o |
| C 1597 | 289 | 7.1 | o | a2 | a1 | a2 | a2 | a2 |

From the times of their occurrence with respect to the star's changes, it is possible to predict the nature of the sources in which identifications may be expected. Thus λλ 4166, 4461.5, and 4521 occur at or near maximum and should be expected in comparatively high-temperature sources, while λλ 3852, 3907, 3938, 4372, and 4579 are evidently of low-temperature origin.

## 8. RADIAL VELOCITIES GIVEN BY THE EMISSION LINES

The hydrogen lines $H\delta$ and $H\gamma$, the iron lines λλ 4202, 4216, 4291, 4308, and 4376, and the magnesium line λ 4571 have been employed for determining the radial-velocity curve of the bright lines. These are the lines used as standards for correcting the wave-lengths of the bright lines. Since they are the strongest and most persistent of the bright lines, they are well suited for measurement over a considerable portion of the star's period. $H\delta$ and $H\gamma$ are the only lines of the group which appear during increasing light and near maximum. During decreasing light, the hydrogen lines are concurrent

with several of the other lines for a large part of the time. There is no evidence that any of the lines give different velocities; in fact, curves for the individual lines show satisfactory agreement with the mean curve based on all eight lines.

These lines are sharp and of superior quality for measurement on our plates. Campbell[1] and Plaskett[2] found that the hydrogen lines at least are not symmetrical and that measurements were affected by the strength of the line on the plate. In practice the micrometer wire has been set upon the tips of the strongly exposed bright lines. This precaution, which is always possible with plates on which the spectrum of the star is narrow, seems to have avoided any systematic error that might have entered because of asymmetry.

TABLE XIV

NORMAL POINTS FOR EMISSION LINES

| Mean Phase | Mean *V* | Mean Phase | Mean *V* |
|---|---|---|---|
| days | km/sec. | days | km/sec. |
| 16 | +47.1 | 148 | +48.9 |
| 49 | 43.2 | 163 | 53.0 |
| 78 | 42.4 | 194 | 52.0 |
| 96 | 41.5 | 297 | 48.0 |
| 116 | 45.1 | 327 | +47.8 |
| 134 | +48.3 | | |

The measures of the whole series of plates have been examined with this in mind, and a special series of plates was taken on January 11, 1925, with exposures ranging from 1 to 35 min.; but no error of this kind seems to be present. Settings can be made on the bright lines with great accuracy. On the whole, it appears that, although the number of lines used is much smaller, the velocity-curve determined from the bright lines is not greatly inferior in accuracy to that determined from the absorption lines.

The velocities found from the eight standard bright lines, with their weights, are given in the last two columns of Table II. They have been combined into the eleven normals given in Table XIV. These are represented by circles with central points in the lower portion of Figure 4. The plates showing bright lines have been

[1] *Astrophysical Journal*, **9**, 31, 1899.

[2] *Journal of the Royal Astronomical Society of Canada*, **1**, 52, 1907.

weighted arbitrarily, as in the case of the absorption lines. The normals are means with nearly equal total weight.

The plot of the normal points shows conclusively that the velocity given by the emission lines is not constant and does not follow the course of the velocities obtained from the absorption lines. Perhaps the most striking, and doubtless an important, feature of the curve is that at the time of minimum light the bright lines give a velocity which agrees closely with that found from the absorption lines. The portion of the bright-line curve showing the rise from least to greatest velocity is well determined on account of the number of lines available during this interval.

TABLE XV

ABSORPTION *minus* EMISSION VELOCITIES

| Phase | a−e | Phase | a−e |
|---|---|---|---|
| days | km/sec. | days | km/sec. |
| 16 | +16.7 | 148 | + 6.1 |
| 49 | 19.1 | 163 | 0.9 |
| 78 | 17.9 | 194 | 0.3 |
| 96 | 17.4 | 297 | 14.0 |
| 116 | 12.2 | 327 | +16.1 |
| 134 | +7.7 | | |

It is unfortunate that other observers have not followed the star to later phases, for it is often possible to photograph the bright lines when it is not practicable to make exposures of sufficient length to secure the continuous spectrum. The principal features of the bright-line curve are, however, supported by the observations of Stebbins[1] on Mira and by those of Merrill[2] for four other long-period variables. In both cases a range of about 8 km/sec. is shown with a minimum velocity at a phase of about 50 days, followed by a sharp rise. Stebbins failed to recognize this variation because of the small amount of material (9 plates) available and the inclusion of a number of lines which are not subject to this variation in velocity.

Standing by itself, the velocity-curve of the bright lines seems to suggest no explanation of its behavior. If, however, the velocity changes are compared with the dark-line curve a clue is unfolded

[1] *Lick Observatory Bulletins*, **2**, 91, 1903.

[2] *Mt. Wilson Contr.*, No. 264; *Astrophysical Journal*, **58**, 237, 1923.

which needs consideration and may be of considerable value in interpreting the phenomena of the emission lines.

In Table XV the differences between the bright-line normal points and the corresponding points on the dark-line curve are given. Figure 5 is a plot of these points.

It will be noted that there are no points between phases 194 and 297 days on account of the weakness and absence of emission lines at that time. Nevertheless, the points seem to represent a curve whose maximum, minimum, and form can be expressed by the elliptic elements

$$K = 9.5 \text{ km/sec.}$$
$$\gamma = +10.8 \text{ km/sec.}$$
$$e = 0.24$$
$$\omega = 311^\circ$$
$$T = 155 \text{ days}$$
$$a \sin i = 42{,}000{,}000 \text{ km}$$

The maximum difference of velocity is found at phase 56 days, the periastron or most rapid velocity change is at 155 days, and at phase 180 days there is practically no difference in velocity between bright and dark lines. The curve through the bright-line normal points in Figure 4 has been derived by correcting the dark-line curve by the differences shown in the curve of Figure 5. The dotted portion of the curve is not covered by observations and it would, perhaps, be well to omit that portion entirely.

## 9. RELATIONSHIP BETWEEN SPECTRAL CHANGES AND LUMINOSITY VARIATION

With Figures 1–7 before us it is possible to obtain a comprehensive idea of the relations between the physical processes taking place in the star, even though we cannot clearly outline the forces which are at work.

The following points appear to be well taken:

1. All physical changes repeat themselves with only slight deviations in succeeding cycles.

2. The spectral type of the star, and presumably the temperature of the absorbing strata, vary with the total luminosity, the earliest type and highest temperature occurring at maximum light. This ap-

plies generally, to different cycles as well as to changes during a single cycle.

3. The appearance and intensity of the emission lines depends essentially on magnitude rather than on phase. It is necessary, however, to distinguish between the ascending and descending branches of the light-curve, which have quite different spectroscopic behavior. For example, in 1917 and 1918, when the light-curves were decidedly different in form, the appearance and intensity of the bright lines seemed to conform more closely to the magnitude changes than to the time elapsed since the preceding maximum. A number of illustrations of this effect may be seen in Table IX where both magnitude and phase are given. The relationship was discovered in an attempt to arrange all the spectrograms in an unbroken sequence.

4. The hydrogen and enhanced iron lines have their maximum intensity of emission when the star is brightest, or shortly after. Certain low-temperature lines appear as bright lines after maximum light and reach their greatest intensity at about magnitude 8.5.

5. The velocity-curve of the absorption lines is similar in shape and phase to the light-curve. It is opposite in phase to the usual Cepheid curve but has a similar eccentricity. This curve is doubtless the fundamental velocity-curve of the lower layers of the star's atmosphere. It is subject to correction owing to the fact that the velocities are integrated over the whole disk. Such a correction would increase the range of variation if the changes are due to pulsation or convection currents.

6. The emission lines of hydrogen and the bright low-temperature iron lines have the same displacement.

7. The velocity-curve of the hydrogen and low-temperature emission lines is more intelligible if referred to the absorption-line curve. At minimum the two curves give equal velocities. The greatest difference in velocity is 56 days after maximum. The bright lines always show an outward velocity with respect to the absorbing strata.

8. The low-temperature bright lines appear when the difference in velocity is greatest and disappear near minimum when the difference is zero.

9. The bright enhanced lines and the lines of silicon (and

indium) have smaller displacements than the hydrogen and low-temperature lines and probably lie at low levels in the atmosphere. They show in less degree the velocity changes indicated by the emission lines of higher level.

### 10. COMPARISON OF OBSERVATIONS AT DIFFERENT CYCLES

The radial velocity from the absorption lines has been measured near maximum by a number of observers during the last twenty-eight years. A comparison of the mean results obtained within thirty days of maximum is given in Table XVI.

TABLE XVI

VELOCITY AT MAXIMUM

| Maximum | Mag. | *V* | No. Plates | Prisms | Observer |
|---|---|---|---|---|---|
| | | km/sec. | | | |
| Nov. 1897..... | 3.7 | +62.0 | 3 | 3 | Lick, Campbell |
| Oct. 1898..... | 2.9 | 62.8 | 2 | 3 | Lick, Campbell |
| June 1902..... | 3.5 | 66. | 1 | 1 | Lick, Stebbins |
| Dec. 1906..... | 3.9 | 65.4 | 2 | 3 | Ottawa, Plaskett |
| Dec. 1906..... | 3.9 | 66.1 | 3 | 3 | Bonn, Küstner |
| Dec. 1906..... | 3.9 | 64.2 | 4 | 3 | Yerkes, Frost |
| Dec. 1906..... | 3.9 | 71. | 2 | 1 | Yerkes, Frost |
| Jan. 1915..... | 3.9 | 70. | 3 | 1 | Yerkes, Frost |
| Jan. 1915..... | 3.9 | 54. | 2 | 1 | Ottawa, Harper |
| Jan. 1915..... | 3.9 | 63.7 | 1 | 1 | Detroit, Merrill |
| Dec. 1915..... | 3.5 | 63.4 | 5 | 4 | Cape, Lunt |
| Dec. 1915..... | 3.5 | 66. | 1 | 1 | Yerkes, Frost |
| Nov. 1916..... | 3.8 | 63.3 | 3 | 1 | Mt. Wilson |
| Sept. 1917..... | 3.6 | 64.1 | 5 | 1 | Mt. Wilson |
| Sept. 1917..... | 3.6 | 66. | 2 | 1 | Yerkes, Frost |
| Sept. 1918..... | 3.6 | 68. | 2 | 1 | Yerkes, Frost |
| Sept. 1918..... | 3.6 | 70. | 1 | 1 | Ottawa, Harper |
| Sept. 1918..... | 3.6 | 59.1 | 1 | 1 | Mt. Wilson |
| Aug. 1919..... | 3.3 | 65.8 | 1 | 1 | Mt. Wilson |
| Aug. 1919..... | 3.3 | 69. | 1 | 1 | Yerkes, Frost |
| June 1920..... | 3.1 | 64.8 | 2 | 1 | Mt. Wilson |
| Mch. 1923..... | 2.8 | 66.2 | 3 | 1 | Mt. Wilson |
| Feb. 1924..... | 4.7 | 62.0 | 4 | 1 | Mt. Wilson |
| Jan. 1925..... | 3.8 | 66.4 | 3 | 1 | Mt. Wilson |
| Jan. 1925..... | 3.8 | +61.3 | 1 | 3 | Mt. Wilson |

These observations may be grouped according to magnitude as in Table XVII.

It is clear that the velocities from the absorption lines do not vary with the maximum magnitude of different cycles to any appreciable extent, and, in fact, are constant within the errors of measurement, as earlier observers have concluded. The velocity-

curves for each season separately show that the whole velocity-curve, as well as the maximum, is very probably constant; but the conclusion does not carry much weight because of the lack of data.

On account of the different methods of measuring the bright lines used by various observers, it has not seemed worth while to include these lines in Table XVI. The Mount Wilson results for the maxima of eight different cycles give the values of the mean velocities of bright $H\delta$ and $H\gamma$ within 30 days of maximum as shown in Table XVIII.

TABLE XVII

VELOCITY AND MAGNITUDE AT MAXIMUM

| Mag. at Max. | $V$ | Wt. |
|---|---|---|
| 2.8–2.9 | +64.3 | 2 |
| 3.1–3.6 | 64.7 | 7 |
| 3.7–3.9 | 64.6 | 10 |
| 4.7 | +62.0 | 1 |

TABLE XVIII

VELOCITIES FROM EMISSION LINES AT DIFFERENT MAXIMA

| Mag. at Max. | $V$ | No. of Plates |
|---|---|---|
| | km/sec. | |
| 2.9–3.1..... | +48.5 | 5 |
| 3.4–3.6..... | 47.9 | 8 |
| 3.7–3.8..... | 47.4 | 7 |
| 4.7..... | +46.3 | 5 |

The tendency for larger values of the velocity to occur at brighter maxima is suggested, but the result may be accidental, since at maximum light only two lines are available for measurement.

From estimates of intensity, we have the approximate relationship of intensity of bright lines to maximum brightness at different epochs as grouped in Table XIX. The values are rounded off from those given in Table IX, and supplemented by a few estimates not included there. It is assumed that the effective exposure for the continuous spectrum is the same for all the plates considered. The results are very rough, but they give an idea of the strengthening of the emission lines with increased brightness at maximum.

The hydrogen lines are thus affected, not only at the time of maximum, but throughout a considerable portion of the particular cycle involved, as is apparent in the two cycles reaching maximum in June, 1920, and March, 1923, when the magnitudes were 3.1 and 2.8, respectively. The strength of $H\beta$ at bright maxima is remarkable. There seems to be some evidence that at times of unusual activity $H\gamma$ is strengthened with respect to $H\delta$. This was certainly true in 1923 (mag. 2.8). One is led to conjecture whether the normal

TABLE XIX

EMISSION-LINE INTENSITIES AT DIFFERENT MAXIMA

| MAG. AT MAX. | INTENSITY | | |
|---|---|---|---|
| | $H\delta$ | $H\gamma$ | $H\beta$ |
| 2.9–3.1 | 65 | 75 | 40 |
| 3.4–3.6 | 50 | 40 | 15 |
| 3.7–3.8 | 45 | 35 | 6 |
| 4.7 | 35 | 30 | 2 |

order of intensities of the hydrogen lines might not prevail at a very bright maximum, such as that recorded by Herschel (mag. 1.5). The spectral type would then be expected to be M1 or M2, and the conditions would perhaps be the same as in those stars of class M1e, such as T Centauri, whose hydrogen lines decrease in strength toward the violet.

Lockyer[1] has collected the published data on this point. Of the 6 maxima fainter than the normal magnitude, 3.5, all have $H\delta$ stronger than $H\gamma$; but for the 6 maxima brighter than 3.5, $H\gamma$ is stronger than $H\delta$ on one occasion and equal to it at two other maxima. One of the latter two is the maximum of 1923 when the Mount Wilson observations indicate that $H\gamma$ was the brighter. Estimates show that for maxima of magnitude 3.5 or brighter, $H\delta$ averages 10 per cent stronger on the scale of Table XIX than $H\gamma$, while for maxima fainter than magnitude 3.5, $H\delta$ is 54 per cent brighter.

The spectral type at maximum varies in a marked degree with the magnitude, as is shown in Table XX.

During the last ten cycles of Mira's variation two extraordinary

[1] *Monthly Notices of the Royal Astronomical Society*, **84**, 563, 1924.

departures from the usual procedure have occurred. First, at the minimum of November, 1922, the star did not fall below magnitude 8.5, which is twice the normal brightness. Second, the maximum of February, 1924, was so feeble that the greatest brightness (mag. 4.7) did not exceed one-third its normal maximum. These remarkable occurrences seem to be due to causes extraneous to the forces which produce normal variations and the minor irregularities accompanying them.

The spectrograms taken at these times are marked by several peculiar features. At the minimum of 1922 the usual conditions seemed to prevail, except that λ 4308 was abnormally weak until

TABLE XX

SPECTRAL TYPE AT DIFFERENT MAXIMA

| Mag. at Max. | Type | Mag. at Max. | Type |
|---|---|---|---|
| 2.9 | M5 | 3.6 | M6 |
| 3.1 | 6 | 3.7 | 7 |
| 3.4 | 5 | 3.8 | 7 |
| 3.5 | 6 | 4.7 | 7 |

September 28, phase 162 days, magnitude 8.2. By November 5, phase 200 days, magnitude 8.5, a great change in the appearance of the spectrum had taken place. The whole series of bright hydrogen and low-temperature lines had disappeared, although only normal changes had taken place in the bands and in the few absorption lines which could be seen. Thirty days later, December 5, phase 229 days, magnitude 8.3, the star had begun to increase in brightness. The spectrum was strong in the region λ 4000–λ 4200 and great numbers of absorption lines were present which would not normally be seen in such strength until two months after minimum. Slow and gradual changes ensued while the star was reaching the maximum of February, 1923, which was the brightest (mag. 2.8) of the last 20 years. No unexpected features were noted after December 5. From a spectroscopic viewpoint the minimum of 1922 seems to have hurried through its changes with extraordinary rapidity. If the time allotted for the changes had been twice as great, the minimum would have passed as a normal one. The light-curve shows

the effect to a lesser degree; the minimum occurred 15 days, and the following maximum, 9 days in advance of the time predicted by mean elements; the preceding decline in brightness was nearly normal to the seventh magnitude; the minimum was flat, and the rise to maximum very steep.

The exceptionally low maximum of February, 1924, is especially interesting because of the appearance at that time of a new and unknown series of bright lines or bands and the presence of broad absorption bands in the region λ 4600–λ 4900, which have not hitherto been noted in any stellar spectrum. These peculiarities may be seen in Plate XX where spectrogram C 2663 is reproduced alongside C 2171, taken March 28, 1923 (mag. 3.2), for comparison.

The most prominent bright band, after correction for the usual velocity of bright lines, has its center at λ 4844.7. Its width is about 7 A. Other strong bright regions less than 1 A in width are found at λλ 4535.5, 4649.15, 4672.76, 4695.40, and 4867.53. These have the appearance of rather wide bright lines, but must be related to the accompanying bands, for there seems to be an absorption band from 5 to 15 A in width to the violet of each of the bright regions. The absorption is not more than one-fourth the intensity of the titanium oxide band at λ 4803, and seems to fade toward the violet, opposite to the titanium bands.

In order to separate the peculiarities of the maximum of 1924 from a normal maximum, Koch microphotometer curves of plates C 2663 and C 2171 were compared. By subtracting the normal curve from the abnormal one, dark bands were disclosed in the regions

| | |
|---|---|
| λλ 4584–4647 (6) | λλ 4722–4758 (2) |
| 4658–4666 (1) | 4760–4803 (6) |
| 4686–4713 (1) | 4815–4842 (2) |

The relative intensity of absorption is given in parentheses. The two strongest bands are affected by titanium absorption, which, at the maximum of 1924, was somewhat greater than normal.

Regions of strengthened emission are indicated at:

λλ 4649–4661 (2)
4669–4695 (4)
4804–4816 (3)
4843–4905 (7)

The band structure may be tentatively identified with that of magnesium hydride, which has been positively recognized in the spectrum of sun-spots, but does not appear in the solar spectrum itself. A comparison of these wave-lengths with Fowler's laboratory spectrum[1] of magnesium hydride shows so many features in common that if there were any positive evidence of the presence of the strong absorption band in the green with its head at λ 5210, the identification would be quite certain. Spectrogram γ 12447 shows a portion of the green region faintly, but beyond λ 5167 the titanium band cuts down the continuous spectrum to a mere shadow. What little evidence this plate affords may be said to be favorable to the identification, but it is hardly conclusive.

These unusual features are found on all six plates from January 12 to March 11, 1924, and doubtless are intimately connected with the cause of the low maximum. It is unfortunate that no spectrograms were taken in the visual region at this maximum in order to show whether there were other absorption bands farther to the red. The weakness of the bright hydrogen lines at the 1922 maximum may be due, in part, to loss of hydrogen taken up by the molecules of the hydride.

The new titanium bands described in Section 4 are especially strong at this maximum.

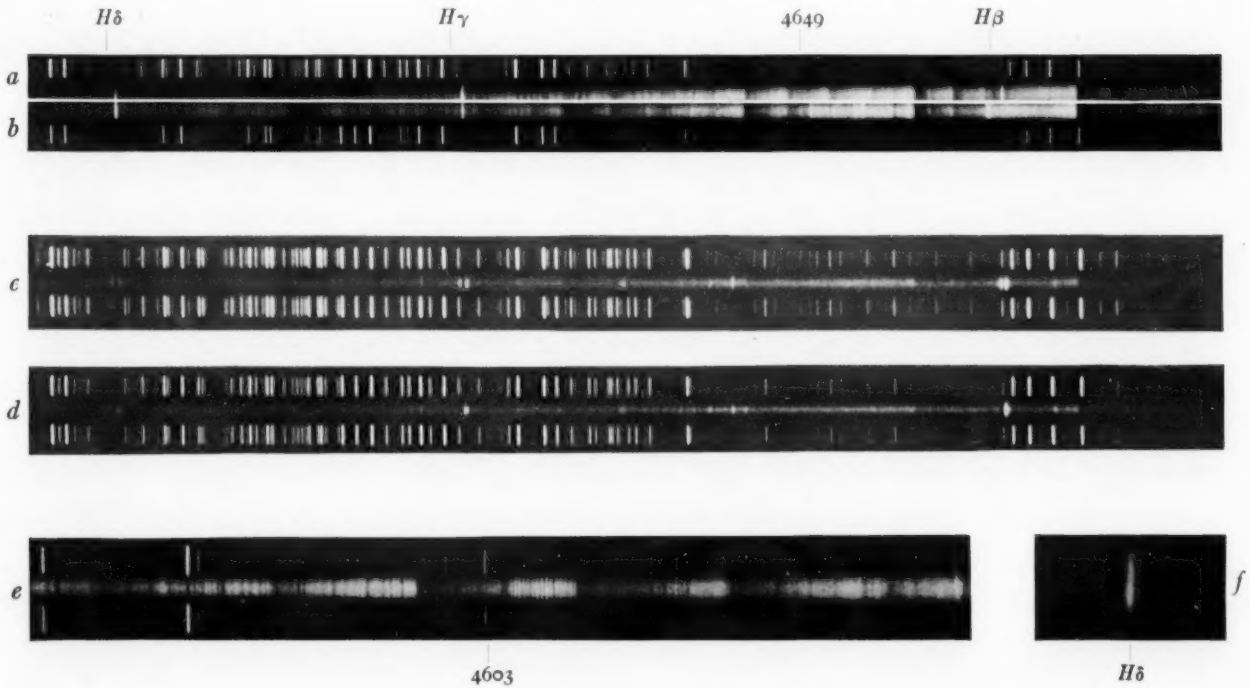
## 11. ABSOLUTE MAGNITUDE, SURFACE BRIGHTNESS, MASS, AND DENSITY

Unfortunately, the distances of the Me stars are not well enough known to permit the calibration of the absolute-magnitude curves of spectral lines. Several of the lines which give acceptable values for the spectroscopic absolute magnitude of giant M stars may, however, be used to estimate the luminosity of *o* Ceti. Those found practicable are λλ 4077, 4215, 4258, and 4490. By using the curves for the ordinary M stars, the absolute magnitude of *o* Ceti at normal maximum is found to be −0.8. The computations of Merrill and Strömberg,[2] and also those of Wilson referred to in the same paper, indicate that the spectroscopic values thus found for Me variables

[1] *Philosophical Transactions of the Royal Society of London*, A, **209**, 478, 1909.

[2] *Mt. Wilson Contr.*, No. 267; *Astrophysical Journal*, **59**, 104, 1924.

# PLATE XX




SPECTROGRAMS OF MIRA AND ITS COMPANION

(*a*) C 2171, March 28, 1923; magnitude 3.2. Normal maximum.
(*b*) C 2663, February 11, 1924; magnitude 4.7. Peculiar faint maximum.
(*c*) C 2482, October 19, 1923. Spectrum of the companion, with bands of Me star, showing hydrogen lines double.
(*d*) C 2989, September 7, 1924. Spectrum of the companion, with bands of Me star, showing violet component of hydrogen lines absent.
(*e*) γ 13185, January 8, 1925. Detailed structure of titanium bands near λ 4600, taken by Sanford with 3 prisms and 18-inch camera
(*f*) γ 13892, November 25, 1925. Triple structure of *Hδ* near maximum, taken with 3 prisms and 40-inch camera.

are too bright by about one-half a magnitude. The corrected absolute magnitude of Mira at normal maximum would be $-0.3$. The corresponding parallax is $0''.017$, which is in reasonable agreement with the best statistical determinations. With this we must be content for the present for, unfortunately, trigonometrical parallaxes of *o* Ceti cannot be used until they have been properly corrected for the effect of the companion in displacing the photographic images. It is probable, however, that in a few years the relative motion of the companion will be sufficient to permit the determination of a dynamical parallax.

The luminosity lines show sufficient change in intensity to indicate clearly that the spectroscopic absolute magnitude becomes considerably fainter as the visual brightness decreases. The lack of suitable reduction curves makes it impossible, however, to evaluate the corresponding absolute magnitude differences.

Using the parallax of $0''.017$ found above and the angular diameter of $0''.056$ measured by Pease[1] with the interferometer, we find the linear diameter to be 354 times that of the sun, or 490,000,000 km (310,000,000 mi.). At normal maximum (mag. 3.5) the average surface brightness per unit area is 7.5 magnitudes fainter than that of the sun.

The giant character of Mira leads us to believe that its mass is rather great, but there seems to be no direct method of determining it at present. The combined and individual masses may ultimately be found from its orbit and its orbital movement with respect to other stars.

The researches of Seares[2] indicate that the mass should be between 1 and 10 times that of the sun. If we take an intermediate value of $5\odot$, the corresponding density for the star is $1.1\times10^{-7}$ as compared with the sun, or about one ten-thousandth that of our atmosphere. The surface gravity is very low, being $4\times10^{-5}$ times that of the sun.

## 12. TEMPERATURE

An estimate of the effective temperature at normal maximum (mag. 3.5, type M6) may be obtained by extrapolation from Abbot's

[1] *Publications of the Astronomical Society of the Pacific*, **37**, 90, 1925.

[2] *Mt. Wilson Contr.*, No. 226; *Astrophysical Journal*, **55**, 202, 1922.

table.[1] He finds from radiometric measures of stellar spectra and comparison with black-body energy curves the values given in Table XXI. The normal maximum temperature of *o* Ceti must therefore be very nearly 2400° K. For the Me stars, many of which are of later type than Mira, Merrill,[2] from a comparative study of the behavior of stellar lines in the electric furnace, has estimated the temperature to be 2200° K.

TABLE XXI

ABBOT'S STELLAR TEMPERATURES

| Star | Type | Temperature |
|---|---|---|
| α Tauri | $K_5$ | 3000° K |
| β Pegasi | $M_2$ | 2850 |
| α Orionis | $M_2$ | 2600 |
| α Herculis | $M_5$ | 2500 |

King[3] has shown that when sufficient oxygen is present the titanium oxide bands appear strongly in the electric furnace from 1900° to 2600°.

Nicholson and Pettit,[4] from thermocouple measures of total radiation, with and without a water cell, find temperatures of 2300° and 1800° K for average maximum and minimum, respectively, during the last four cycles. For the same period they observed an average total energy change from maximum to minimum of 1.1 magnitudes.

If we assume that the change in visual brightness varies as the radiation at λ 5750, which is the maximum of the visibility curve for these stars, we have from Planck's law of energy distribution,

$$\frac{B_1}{B_2} = \frac{e^{\frac{C_2}{\lambda T_2}} - 1}{e^{\frac{C_2}{\lambda T_1}} - 1}$$

[1] *Mt. Wilson Contr.*, No 280; *Astrophysical Journal*, 60, 105, 1924.

[2] *Mt. Wilson Contr.*, No. 265; *Astrophysical Journal*, 58, 200, 1923.

[3] *Mt. Wilson Contr.*, No. 114; *Astrophysical Journal*, 43, 341, 1916; *Publications of the Astronomical Society of the Pacific*, 36, 140, 1924.

[4] Unpublished.

where $B_1$ and $B_2$ are the visual luminosities at maximum and minimum, respectively, and $T_1$ and $T_2$ the corresponding temperatures. Taking $C_2 = 14300$, $T_1 = 2300°$ K, and $T_2 = 1800°$ K,

$$\frac{B_1}{B_2} = 3.3 \text{ mag.}$$

By the fourth-power law these temperatures indicate a difference in energy of 1.1 magnitudes.

Thus, according to black-body laws, a change in total energy of 1.1 magnitudes at these temperatures corresponds to a visual range of 3.3 magnitudes. The average visual variation of Mira for the same period is 5.3 magnitudes. The difference between the observed visual range and the amount calculated from the radiometric measures is 2.0 magnitudes. This difference can be attributed to the excessive absorption of the titanium oxide bands at minimum.

Suitable plates for photometric testing of the energy in different spectral regions at maximum and minimum are not available, but an idea of the absorption produced by the titanium bands may be gained from a comparison of Mira at maximum, when the type is M6, with a K5 giant star, in which the bands are not noticeably present.

For this test, spectrograms of Mira and Boss 646 were made with the 100-inch telescope on December 6, 1925, when the stars were at the same zenith distance. The regular Cassegrain spectrograph was employed, but to avoid troubles from atmospheric dispersion the slit was opened to one-half a millimeter. Ilford Panchromatic plates were used and developed together with dilute Rodinal. A strip at the edge of each plate was exposed to a Mazda lamp through a graduated photographic wedge whose energy transmission was calibrated by Mr. Pettit. Koch microphotometer curves of the spectra and the graduated strip were made. Areas under the curve were measured and reduced to relative energy by the wedge constants. In this way it is possible to compare the relative energy in the two stars for any part of the spectrum between λλ 4100 and 6800.

A rough estimate of the absorption of the titanium bands can now be made by comparing the energy distribution in *o* Ceti at maximum with that of Boss 646. If we equalize the energy of the

two stars for the limited region λλ 4100–4300, where the bands do not show, by applying the proper factor to the measured energy of one of the stars, then the remaining portions of the two spectra, comprising practically the whole visual region, will show unequal energies arising from the difference in temperature and from titanium absorption. A comparison of black-body energy-curves for temperatures of 3000° and 2500° K shows that the visual region of the cooler star ought to give 2.0 times as much energy as that of the

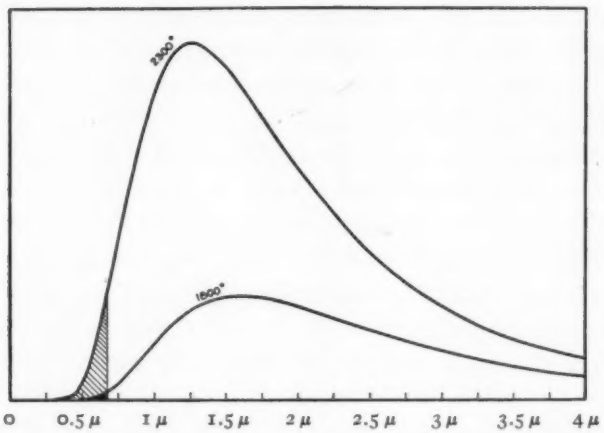



FIG. 8.—Black-body curves for 1800° and 2300° K, showing a difference in total radiation of 1.1 magnitudes. The shaded portions represent the region λλ 3000–6700 and indicate the much larger difference in the visible radiation, which at these temperatures is only a small fraction of the total radiation.

hotter star when the regions from λλ 4100–4300 are brought to equality. Measures of the microphotometer curves show, on the contrary, that, when the violet regions are equalized, *o* Ceti shows less energy in the visual region (λλ 4300–6700) than Boss 646 by a factor of 3.0. It is evident that the titanium oxide absorption is represented by a factor of 6.0, or 1.9 magnitudes. Inspection of plates taken at minimum indicates that the increase in strength of the bands at minimum is of the same order, and that the discrepancy of 2.0 magnitudes between visual and radiometric measures may be fully accounted for in this way. The greatest outstanding uncertainty is due to our lack of knowledge of the extension of the titanium bands into

the infra-red. A study of the actual amount of titanium absorption at different phases would be a valuable contribution to the problem of the star's variation.

The overlapping of bands is so general at low temperatures that nearly all of the continuous spectrum in these regions is reduced more or less in intensity. The more refrangible rays on the violet side of λ 4200 are not absorbed and, as Stebbins[1] has pointed out, are thus relatively stronger at minimum.

Figure 8 shows the calculated energy-curves at maximum and minimum, for temperatures 2300° and 1800° K, and a difference in total radiation of 1.1 magnitudes. It is instructive to note the small percentage of energy falling within the visual region and the large effect of temperature change upon the amount of visible light.

The question of the applicability of black-body laws to a star of this character is still open. There may be complications connected with the nature of its variability and its bright lines, but it seems very probable that when the effects of bands and atmospheric absorption are taken into account, the radiation given out will closely follow the normal energy-curve.

## 13. THE DISCOVERY OF THE VISUAL COMPANION

Although the emission line of $H\beta$ belonging to the companion appears on spectrograms of *o* Ceti taken in 1918, it was not until the minimum of January, 1920, that it was possible to obtain spectra at minimum and to appreciate the extraordinary character of the hydrogen lines and the peculiar effect of the superposition of the continuous spectrum of the companion on the banded spectrum of the variable star. During the next two years minima occurred during the winter months and little progress was made in solving the problem. Although Professor Barnard failed to resolve the star under bad conditions of seeing, the impression of duplicity gained in strength. The position angle and distance of the source of the peculiar spectrum were roughly estimated from spectrograms taken with the slit in different position angles to be 135° and 0".3 respectively.[2]

[1] *Lick Observatory Bulletins*, 2, 90, 1903.

[2] *Publications of the American Astronomical Society*, twenty-ninth meeting, p. 15, 1922; *Popular Astronomy*, 31, 237, 1923.

In 1923 the star seemed to be elongated on the slit of the spectrograph. At our suggestion, Dr. Robert G. Aitken examined the star with the 36-inch refractor of the Lick Observatory on October 19 and found it to be an easy double; position angle, 130°.3; distance, 0".90; magnitude of the companion, about 9.8.[1] At the minimum of 1924 there was "little or no evidence of change."[2]

The companion was noticeably fainter in 1925. On the spectrograms taken in July the continuous spectrum of the companion was relatively weak as compared with the two preceding years. On August 25, during a visit to Mount Hamilton, Drs. Aitken and Moore showed the writer the star with the 36-inch refractor. The companion was at least 1½ magnitudes fainter than the primary and was not easily seen under the rather poor conditions of seeing then prevailing. They pointed out that, under similar conditions, it would have been easy to observe at the two preceding minima. The September spectrograms show no trace of the continuous spectrum of the companion and only faint indications of the bright hydrogen lines. Unquestionably, the companion is also a variable star of long or irregular period. It was, doubtless, too faint to have been seen by Aitken in 1903 and Doolittle in 1905.

The maximum brightness seems to have been reached in 1922 or 1923 and to have fallen off rapidly in 1925. A number of the stars of its spectral type with bright lines have been found to be irregular variables.

### 14. THE SPECTRUM OF THE COMPANION

During minima of *o* Ceti it has been possible to secure spectrograms of the companion with very little interference from the brighter star. The outstanding features of the spectrum are: the hydrogen lines, made up of very strong emission components with absorption centers; the strong continuous spectrum whose intensity distribution resembles that of the late B-type stars; the bright bands of helium and ionized calcium; and the fairly sharp but weak bright lines of ionized iron.

No absorption lines are seen except those accompanying the hydrogen series. The spectral type is probably about B8. This de-

[1] *Publications of the Astronomical Society of the Pacific*, **35**, 323, 1923.

[2] *Ibid.*, **36**, 296, 1924.

termination is based on the continuous spectrum, on the fact that bright helium is present, and on analogy with similar stars in which helium absorption lines make their classification possible.

H.D. 50138 has similar hydrogen lines, while H.D. 161114[1] has bright lines of ionized iron. The bright bands of helium and of H and K of calcium seem to be unique. They are 4 or 5 A in width and usually are not suitable for accurate measurement. They have approximately the same displacement as the sharp bright lines of iron. The helium lines, λλ 4026, 4388, 4471, 4713, $D_3$, and probably 4922 and 5016, are present.

The hydrogen series has been observed from $H\alpha$ to $H\epsilon$. The lines fall off in intensity toward the violet, as is usually the case with bright-line stars other than the Me variables. $H\alpha$ has been seen on three photographs, but its exact character and strength cannot be determined on account of overexposure in that region. $H\epsilon$ is faintly seen on a few plates. The other lines, $H\beta$, $H\gamma$, and $H\delta$, are much alike in structure and, on account of their great intensity, are easily observed with moderate exposures. $H\gamma$ is much stronger with respect to its continuous spectrum than $H\gamma$ of the Me star at its maximum intensity. $H\beta$ is about three times the intensity of $H\gamma$, and $H\delta$ is correspondingly weaker.

The hydrogen lines are made up of three parts: a strong emission component about 4 A in width, displaced about 2 A to the red from its normal position after allowing for the velocity of the star; a strong absorption line 2 A in width, displaced to the violet about 1 A; and a violet emission component of variable strength lying 3 A to the violet of the normal position of the hydrogen lines.

The hydrogen lines, at times, resemble those of P Cygni. The violet components of $H\beta$, $H\gamma$, and $H\delta$ are nearly equal in intensity, showing only a slight strengthening from violet toward the red. As a result the violet component is stronger with respect to the red in $H\delta$ than at $H\beta$. At some minima the violet component is very faint, while in 1923 it was comparable with the red component. The relative maximum strength of the violet component of $H\beta$ at the successive minima from February, 1920, to August, 1925, was 1, 5, 5, 5, 10, 2, 2. Further, the violet component is proportionately brighter near

[1] Merrill, *ibid.*, p. 225, 1924.

the minimum phase of the Me star. Its maximum strength is found in every case on plates taken between the phases 205 and 230 days. The intensity increases up to the time of minimum and decreases

TABLE XXII

INTENSITIES OF THE VIOLET COMPONENT OF $H\beta$

| Phase | Intensity | Phase | Intensity |
|---|---|---|---|
| days | | days | |
| 167..... | 1 | 201..... | 4 |
| 182..... | 2 | 204..... | 5 |
| 197..... | 2 | 230..... | 2 |
| 198..... | 2 | 256..... | 0 |

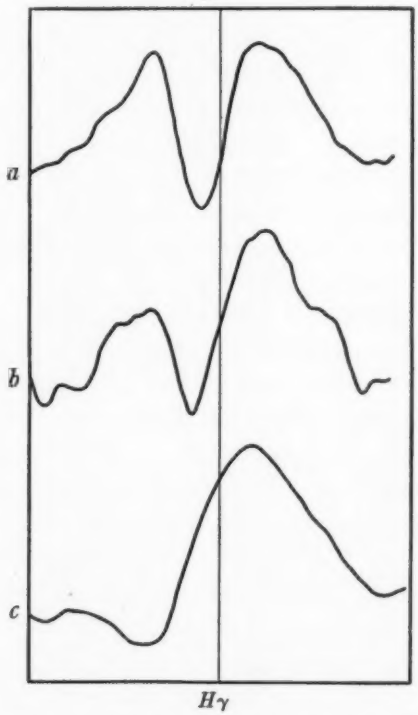



FIG. 9.—Curves drawn from thermocouple deflections showing photographic intensities of $H\gamma$ in the spectrum of the companion of Mira. (*a*) C 2482, October 19, 1923; phase 218 days. (*b*) C 1494, December 14, 1921; phase 204 days. (*c*) C 1547, February 5, 1922; phase 257 days. The vertical line represents the normal position of $H\gamma$. Scale, 3 mm = 1 A.

thereafter. For example, in 1921 the estimated intensities were as given in Table XXII.

The intensity thus appears to vary with the brightness of the companion and inversely with that of Mira.

There is no certain indication of variation in the position or intensity of the red emission component. Figure 9 illustrates the $H\gamma$ line at three different phases. The curves are formed from thermocouple deflections taken with a narrow slit at intervals in the region of $H\gamma$ and give accurate relative intensities of the photographic blackening in that region of the spectrum. The vertical line indicates

TABLE XXIII

EMISSION LINES OF COMPANION

| λ (Rowland) | λ (I.A.) | λ (Rowland) | λ (I.A.) |
|---|---|---|---|
| 4233.33..... | .16 | 4520.40.... | .24 |
| 4302.35..... | .18 | 4522.80.... | .64 |
| 4303.34..... | .18 | 4534.34.... | .17 |
| 4351.93..... | .77 | 4549.64.... | .48 |
| 4416.98..... | .81 | 4556.06.... | .90 |
| 4489.35..... | .21 | 4584.02.... | .84 |
| 4491.57..... | .41 | 4924.11.... | .92 |
| 4508.45..... | .29 | 5018.63.... | .44 |

the position of $H\gamma$ displaced toward the red to allow for the velocity of the star as determined from the bright lines.

It will be noted that the absorption line is so strong that, in each case, the curve falls well below the level of the neighboring continuous spectrum. The absorption seems to fall farther to the violet as the violet component becomes weaker, but this effect may be apparent rather than real.

The whole band extends about equal distances on either side of the normal position of $H\gamma$, but the absorption line is situated well to the violet of the center.

Emission lines of ionized iron, as given in Table XXIII, have been identified. Many of these lines are very faint on our spectrograms. As they are also of poor quality for measurement, the results show a wide range and should be given small weight. Table XXIV shows the velocities resulting from the identification given above.

The mean velocity of +51.0 km/sec. is nearly that of both dark

and emission lines of the Me star at minimum. During the three years covered by these observations there was probably no change in velocity.

The bright lines comprise most of the stronger lines of ionized iron in the blue region of the spectrum. Lines belonging to other elements, if present, are too faint to be seen.

If the maximum apparent magnitude of the companion star is 9.8 and the absolute magnitude of the principal star is −0.3 at

TABLE XXIV

VELOCITY OF COMPANION FROM BRIGHT LINES

| Plate | No. Lines | *V* |
|---|---|---|
| | | km/sec. |
| C 2482..... | 3 | +53.3 |
| 2930..... | 8 | 52.3 |
| 2989..... | 13 | 52.0 |
| 3037..... | 5 | 47.8 |
| 3406..... | 8 | 53.9 |
| 3410..... | 3 | 45.6 |
| Weighted mean..... | | +51.0 |

normal maximum (mag. 3.5), the absolute magnitude of the companion is +6.0. It must be considered as an early-type dwarf star, and is the only one thus far found to have a bright-line spectrum.

## 15. DISCUSSION

It is not the purpose of this paper to attempt a theory of the variation of the stars of the Mira type. The problem of the underlying cause of the periodic physical convulsion which produces the variation of these stars is one which will ultimately yield to theoretical attack when sufficient observational material is at hand. It may be well to consider briefly the bearing of recent study on the question.

If the velocity-curve shown by the absorption lines can be interpreted as a spherical pulsation of the star, it would mean a considerable change in the size of the disk and a corresponding variation in the total luminosity. The largest size would be during the star's increasing light and the smallest on the decreasing branch. At maximum and minimum it would have intermediate dimensions. The

semi-variation of radius given by the elements of the orbit of the dark lines is 26,200,000 km, which is a pulsation of about 11 per cent about a mean position. Since the velocities are integrated over the whole disk of the star, the actual changes would be about one-third greater.[1] This amount is comparable with the relative change found for Cepheids and is a much smaller percentage variation than that found in α Orionis by interferometer measures of diameter. It appears doubtful, indeed, whether the full change, which would produce a variation in brightness of one-half a magnitude or more due to variation in the size of the disk, actually occurs. Such a distortion of the light-curve should be detected in the visual curve and will certainly be noticed, if present, in the bolometric curve. In case the spherical pulsation is disproved, alternative explanations may be sought in non-spherical pulsations or atmospheric currents such as have recently been considered in the case of the sun.

It is not possible to admit of a two-body system in explanation of the radial velocity changes, since, even with the most favorable inclination and masses, the orbit of the secondary star would be well within the limit of the known dimensions of the primary.

The emission lines themselves play little or no part in the light variation, for the total light given out by bright lines is less than 1 per cent of the total visual light. They are, however, of the greatest interest from a physical standpoint. A thorough knowledge of their origin and behavior might open up the whole problem and would shed much light on the question of stellar atmospheres under conditions of extremely low density and gravity.

The emission lines of hydrogen and the low-temperature lines of iron and magnesium give the same velocities, irrespective of the time of their first appearance. Thus, they seem to originate in a moving stratum or shell, rather than in explosions or prominences shot out at intervals from the atmosphere. Even the emission lines of silicon and ionized iron, which are produced under conditions of greater excitation, seem to share the same velocity changes in a less degree.

Except at the time of minimum light, the movements of this

[1] Shapley and Nicholson, *Proceedings of the National Academy of Sciences*, **5**, 422, 1919.

shell, as recorded by the emission lines, are always outward with respect to the reversing layer where the absorption lines are produced. The hydrogen lines appear with the rise in temperature and show increasing velocity until well after maximum. The low-temperature lines appear after maximum when the temperature has fallen sufficiently, but do not find favorable conditions for excitation during the time of rising temperature. When the temperature has dropped to minimum all bright lines disappear for a time.

The location of the emitting layer relative to the absorbing layer cannot be determined conclusively. Emission is usually to be expected at the higher levels, but the behavior of $H\epsilon$ and several of the ultra-violet hydrogen lines suggests an overlying layer or diffuse cloud of absorbing calcium and iron. Possibly the peculiar distribution of intensities found among the hydrogen lines having wavelengths longer than $H\epsilon$ may also be accounted for by absorbing hydrogen.

It should be kept in mind that we see the integrated light from the whole disk of the star and that lines which are bright at the center may or may not be bright at the limb; and, further, that variation of the darkening at the limb would tend to mask luminosity changes.

Recognition of the temperatures involved at maximum and minimum connects the problem of the star's variation more closely with that of other types of physical variation of stars. For, taking into account the fact that a large loss of light in the visual region results from a small change in bolometric magnitude at these temperatures and allowing for the increasing absorption of the titanium oxide bands, we find that variation in energy arising from internal causes is of the order of one magnitude. This leads to the conclusion that the physical changes may be traced to sources similar to those prevailing in the hotter variable stars and that the whole group of physical variables may be found to form a sequence from the shortest to the longest periods.

Temperature estimates have been derived from studies of the spectra and, more recently, have been amply confirmed by radiometric observations. The agreement of these results indicates that the temperatures are known within about 5 per cent.

The important part played by the titanium bands is evident upon inspection of spectrograms taken at different phases of the star's variation. General absorption may also be present, but it seems clear that its effect must be small as compared with the titanium absorption.

The companion is a most unusual star. It will be interesting to observe its future variation in brightness, as well as its motion, and to confirm the influence of the principal star on its emission lines.

It is a pleasure to acknowledge the assistance rendered by many members of the Observatory staff during the progress of this study. All the members of the spectroscopic department have taken part in the observations, measurements, or computations. Mr. Ellerman has reproduced photographs for study and prepared the illustrations. Miss Ware has made the microphotometer curves. Messrs. Nicholson and Pettit have assisted very greatly in the sections concerning temperature and energy distribution. Mr. King has supplied laboratory plates which have helped in the solution of several puzzling questions. Mr. Merrill has given advice on many occasions. Mr. Adams has shared in much of the work, especially in the earlier part of the investigation covered by our joint papers, and has continued his interest throughout.

MOUNT WILSON OBSERVATORY
March 1926

# ON THE TRANSMISSION OF RADIATION THROUGH AN ABSORBING MEDIUM IN MOTION, WITH APPLICATIONS TO THE THEORY OF SUN-SPOTS AND SOLAR ROTATION[1]

By SVEIN ROSSELAND[2]

ABSTRACT

*Transmission of radiation through an absorbing and emitting medium.*—The theory is developed in a general way so as to include cases in which the medium is in motion and the state not stationary in time (§§ 1–5). In §§ 1–4 the properties of the radiation field alone are studied, and in § 5 the equation for the combined flow of material and ethereal energy are given. In § 6 the preceding theory is applied to sun-spots. It is assumed that the energy of solar radiation proceeds mostly from the sun's interior, and that in the spots this uniform stream of radiation is modified by convectional currents. This picture of the phenomenon leads to a simple formula connecting the mean density, mean vertical velocity, and mean vertical thickness of a spot. The theory leads to values of the thickness and velocities corresponding to the observed reduction of temperature in spots which are reasonable, but lack of observations prevents a more rigorous test.

In the last section the theory of solar rotation is discussed. It is emphasized that the equatorial acceleration of the sun may be intimately connected with the manner in which the energy of solar radiation is generated, and that pre-existing theories of the evolution of rotating stars need revision in order to take account of internal motions other than a rigid rotation. Friction being neglected, it is shown by a simple example how the source of energy determines the variation of angular velocity parallel to the axis of rotation, the variation perpendicular to this direction remaining undetermined to this degree of approximation.

1. In all theoretical work on the transfer of energy by radiation through an absorbing medium it is assumed that the medium is at rest. This assumption is sufficiently general to give valuable information about the large-scale structure of the stars. When it comes to the detailed explanation of the phenomena which occur in stellar atmospheres, however, it is obvious that the simple theory of radiative equilibrium is conceived on too narrow a scale, and the development of the theory in its most general aspect, allowing for the motion of the medium as well as for variations in the radiation field with time, becomes a necessity for astrophysical theory.

In the familiar case of the sun the phenomena most urgently calling for a discussion of this kind are the spots. How can a sun-spot retain for months a temperature which is lower than that of

[1] *Contribution from the Mount Wilson Observatory*, No. 313.

[2] Fellow of the International Education Board, 1924–1926.

the surrounding solar surface by a thousand degrees? The most probable answer to this question, on the basis of ordinary physical facts, seems to be that somewhere in the spot matter is in motion upward, and that the expansion of the gases consequent to this motion is sufficient to reduce systematically the flow of radiant energy which would characterize a state of hydrostatic equilibrium. Observations on the reduction of temperature in sun-spots below that of the undisturbed radiating surface contain, therefore, a store of implicit information on the general nature of the vertical motion in spots.

When we study stellar atmospheres in general a number of problems turn up, such, for example, as the Cepheids and long-period variables, and all kinds of spectral variability not directly due to duplicity, the solution of which will probably require the consideration of transmission of radiation through moving media. In what follows, special stress will be placed upon the development of the general theory, while the application to stellar problems, which is by far the more difficult task, will be more in the nature of examples illustrative of this theory.

2. Consider any medium which is capable of emitting and absorbing radiation. Let $E_\nu d\nu$ denote the energy of radiation contained in the frequency interval $\nu$ to $\nu+d\nu$, which is emitted per unit time, volume, and solid angle from the matter at the point $x, y, z$ at the time $t$. Further, let $\varkappa_\nu$ denote the absorption coefficient of the matter at the same point and time, and in the same frequency interval, and let this be defined in such a way that $\varkappa_\nu ds$ is the fractional reduction of a linear beam of radiation in the given frequency interval in traversing the distance $ds$ through the medium in the immediate vicinity of the point in question.

The radiation field will be described statistically by a function $I_\nu$ which is defined by the requirement that

$$I_\nu \, d\nu \, dw \, d\sigma$$

is the energy of radiation in the frequency interval $d\nu$, directed within the elementary solid angle $dw$, which per unit time passes perpendicularly through the surface element $d\sigma$. Thus $I_\nu$ will be a function of $x, y, z, t, \nu$ and the direction cosines of the normal to $d\sigma$

counted in the direction of the flow of radiation. The function $I_\nu$ will occasionally be referred to as the *intensity* of the radiation field. Consider a small fixed cylinder with $d\sigma$ as a base, of height $ds$, and with its axis in the direction of $I_\nu$. The amount of radiation which in the time interval $t$ to $t+\tau$ enters the cylinder through $d\sigma$ is

$$d\nu\, d\sigma\, dw \int_t^{t+\tau} I_\nu\, dt\,.$$

If no absorption or emission of radiation took place within the cylinder, this same amount would escape through the top of the cylinder in the time interval $t+ds/c$ to $t+ds/c+\tau$. The radiation in question will, however, be weakened by absorption to an amount

$$d\nu\, d\sigma\, dw\, ds\, x_\nu\, I_\nu\, \tau$$

correct to the first order of the small quantity $ds$, and $\tau$, and reinforced through emission by the amount

$$d\nu\, d\sigma\, dw\, ds\, E_\nu\, \tau$$

which is correct to the same degree of approximation. The equation expressing the balance of energy in the field of radiation takes the form

$$\int_t^{t+\tau} I_\nu\, dt = \int_{t+ds/c}^{t+\tau+ds/c} \left(I_\nu + \frac{\partial I_\nu}{\partial s}\, ds\right) dt - (E_\nu - x_\nu\, I_\nu) ds\, \tau\,.$$

Developing the integrals to the first order in $\tau$ and $ds$, we obtain the final relation

$$\frac{\partial I_\nu}{\partial s} + \frac{1}{c}\frac{\partial I_\nu}{\partial t} + x_\nu\, I_\nu = E_\nu\,. \tag{1}$$

In order to apply equation (1) to actual problems, $x_\nu$ and $E_\nu$ must be known. These functions regulate the interchange of energy between the radiation field on one hand and the field of matter on the other. The solution of equation (1) involves, therefore, the simultaneous solution of all equations which determine the state of the material field. Before entering upon this latter question we shall consider the problem of the integration of (1), assuming $x_\nu$ and $E_\nu$ to be known functions of the co-ordinates and the time.

Consider first the case in which the state of the system is steady, i.e., in which $I_\nu$, $x_\nu$, $E_\nu$ are independent of the time. Assume that we wish to calculate $I_\nu$ at the point $x$, $y$, $z$, and for direction cosines, $\alpha$ $\beta, \gamma$. We then draw a straight line in the direction $\alpha$, $\beta, \gamma$ and specify the position of the point $x$, $y$, $z$ on this line by a linear co-ordinate $s$. Equation (1) then becomes an ordinary linear differential equation of the first order in $s$. Putting

$$\tau_{AB} = \int_A^B x_\nu \, ds \,, \tag{2}$$

the integration being carried out on the straight line on which $s$ is the co-ordinate and from the point $A$ to the point $B$, we may write equation (1) in the form

$$e^{-\tau_{AB}} \frac{\partial}{\partial s} (I_\nu \, e^{\tau_{AB}}) = E_\nu \tag{3}$$

where the point $B$ has been assumed to be variable while $A$ is a fixed point. Multiplying (3) by $e^{\tau_{AB}}$ and integrating from any point $C$ to a point $D$, we obtain as the integrated form of (1)

$$(I_\nu)_D = (I_\nu)_C e^{-\tau_{CD}} + \int_C^D (E_\nu)_B e^{-\tau_{BD}} \, ds_B \tag{4}$$

where we have made use of the identical relations

$$\tau_{AB} + \tau_{BC} = \tau_{AC} \,, \text{ etc.}$$

Equation (4) might have been written down directly, as it expresses only the fact that the intensity of radiation at $D$ is made up of the intensity at $C$ and the intensity of the radiation emitted from the layer between $D$ and $C$, allowance being made for the absorption, which in both cases takes place on the passage from the point of origin to $D$.

3. Let us now proceed to the solution of (1) for the case in which the term involving differentiation with respect to the time is retained. Put $s = x$, $ct = y$, and $I_\nu = z$, and assume that the final solution will be given in the form

$$V(x, y, z) = 0 \,.$$

Equation (1) then transforms into

$$\frac{\partial V}{\partial x}+\frac{\partial V}{\partial y}-(x_\nu z-E_\nu)\frac{\partial V}{\partial z}=0 \tag{5}$$

which expresses the fact that the vector whose components are 1, 1 and $-(x_\nu z-E_\nu)$ is contained in surfaces $V=$const. The stream lines of this vector are given by

$$\frac{dx}{1}=\frac{dy}{1}=-\frac{dz}{x_\nu z-E_\nu}\,. \tag{6}$$

Let

$$f_1=\text{const.}\,;\qquad f_2=\text{const.}$$

be the integrals of (6). Then

$$V=\Phi(f_1, f_2)=0 \tag{7}$$

constitutes the general solution of (5), $\Phi$ being an arbitrary function[1] of $f_1$ and $f_2$. In the present case it is easy to find the integrals $f_1$ and $f_2$ of the ordinary differential equations (6). The first equation gives directly

$$f_1=x-y\,. \tag{8}$$

The second equation then reduces to

$$\frac{dz}{dx}=-x_\nu(x,\ x-f_1)z+E_\nu(x,\ x-f_1)\,. \tag{9}$$

This is an ordinary linear differential equation having the solution

$$f_2=ze^{\int^x x_\nu(p,\ p-f_1)dp}-\int^x E_\nu(q,\ q-f_1)e^{\int^q x_\nu(p,\ p-f_1)dp}dq\,. \tag{10}$$

Introducing these values of $f_1$ and $f_2$ in (7), solving for $z$, and introducing the original notation we obtain the generalized equivalent of (4)

$$I_\nu=\phi(s-ct)e^{-\int^s x_\nu(p,\ p-s+ct)dp}+\int^s E_\nu(q,\ q-s+ct)e^{-\int_q^s x_\nu(p,\ p-s+ct)dp}dq \tag{11}$$

[1] See, for instance, E. Goursat, *Cours d'Analyse Mathématique*, 2 (Paris 1915), 551.

where $\phi$ is an arbitrary function of one argument, and where the constant limits of integration are left arbitrary. We may revert to (4) by assuming the left-hand side of (11) to be independent of the time. The arbitrary function must then reduce to a constant, since it contains $s$ only in the combination $s-ct$, and by a proper adjustment of the limits of integration the two solutions become identical.

4. *Flux of radiation.*—In order to study the resultant streaming of radiant energy in the field it is convenient to introduce a certain vector $\boldsymbol{F}_\nu$ having the rectangular components $F_{\nu x}$, $F_{\nu y}$, $F_{\nu z}$, defined by the condition that $(\boldsymbol{F}d\boldsymbol{\sigma})$ is the net amount of the radiant energy in the frequency interval $d\nu$ which per unit time passes through the fixed surface element $d\boldsymbol{\sigma}$.[1] The vectorial nature of $\boldsymbol{F}_\nu$ may be inferred from the fact that $(\boldsymbol{F}_\nu d\boldsymbol{\sigma})$ necessarily has the form[2]

$$(\boldsymbol{F}_\nu\, d\boldsymbol{\sigma}) = A\, d\sigma_{yz} + B\, d\sigma_{zx} + C\, d\sigma_{xy} \qquad (12)$$

[1] Vectors and tensors will be denoted by heavy type, and the following symbolism from the vector calculus will be used: Let $\boldsymbol{a}$, $\boldsymbol{b}$, denote any two vectors with rectangular components $a_x$, $a_y$, $a_z$, while $\varphi$ is an invariant. We then write

Inner product of $\boldsymbol{a}$ and $\boldsymbol{b} = (\boldsymbol{ab}) = a_xb_x + a_yb_y + a_zb_z$ = invariant,

Vector product of $\boldsymbol{a}$ and $\boldsymbol{b} = [\boldsymbol{ab}]$ = vector with components $a_yb_z - a_zb_y$; $a_zb_x - a_xb_z$; $a_xb_y - a_yb_x$,

Differential vector operator $\nabla$ = vector with components $\frac{\partial}{\partial x}$; $\frac{\partial}{\partial y}$; $\frac{\partial}{\partial z}$; consequently

$$\nabla\varphi = \text{vector: } \frac{\partial\varphi}{\partial x};\ \frac{\partial\varphi}{\partial y};\ \frac{\partial\varphi}{\partial z},$$

$$\text{div } \boldsymbol{a} = (\nabla\boldsymbol{a}) = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z} = \text{invariant},$$

$$\nabla^2\varphi = \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} = \text{invariant}.$$

Invariant differential operator $(\boldsymbol{a}\nabla) = a_x\frac{\partial}{\partial x} + a_y\frac{\partial}{\partial y} + a_z\frac{\partial}{\partial z}$,

$$\text{curl } \boldsymbol{a} = [\nabla\boldsymbol{a}] = \text{vector: } \frac{\partial a_z}{\partial y} - \frac{\partial a_y}{\partial z};\ \frac{\partial a_x}{\partial z} - \frac{\partial a_z}{\partial x};\ \frac{\partial a_y}{\partial x} - \frac{\partial a_x}{\partial y}.$$

Let $\boldsymbol{A}$ stand for a tensor of second rank with components $A_{xx}$, $A_{xy}$, etc. . . . . $A_{zz}$, while $\boldsymbol{a}$ is a vector. Then we define as inner product of $\boldsymbol{A}$ and $\boldsymbol{a}$ a vector with rectangular components $A_{xx}a_x + A_{xy}a_y + A_{xz}a_z$; $A_{yx}a_x + A_{yy}a_y + A_{yz}a_z$; $A_{zx}a_x + A_{zy}a_y + A_{zz}a_z$, which we denote by $(\boldsymbol{aA})$. Taking for $\boldsymbol{a}$ the differential operator $\nabla$, we obtain a vector which we call div $\boldsymbol{A}$, the components of which are $\frac{\partial}{\partial x}A_{xx} + \frac{\partial}{\partial y}A_{xy} + \frac{\partial}{\partial z}A_{xz}$, etc.

[2] See, for example, Carslaw, *Mathematical Theory of the Conduction of Heat in Solids* (London, 1921).

where $A$, $B$, $C$ are certain coefficients and $d\sigma_{yz}$, $d\sigma_{zx}$, $d\sigma_{xy}$ are the projections of the surface element $d\sigma$ on the $yz$, $zx$, and $xy$ planes of coordinates, respectively. Integrating this expression over a closed surface and transforming the surface integral into a volume integral by Gauss' theorem, we find that

$$\frac{\partial A}{\partial x}+\frac{\partial B}{\partial y}+\frac{\partial C}{\partial z}$$

is an invariant. Consequently $A$, $B$, $C$ form the components of a vector, and the right-hand side of (12) is the scalar product of two vectors. Thus $\boldsymbol{F}_\nu$ is a vector with components $F_{\nu x}=A$, $F_{\nu y}=B$, $F_{\nu z}=C$, and this vector will be referred to as the *flux of radiation*.

From the definition of $\boldsymbol{F}_\nu$ we have

$$(\boldsymbol{F}_\nu d\boldsymbol{\sigma})=d\sigma\int I_\nu\cos\widehat{I_\nu d\sigma}\,dw\,,$$

the integration being extended to $4\pi$ of solid angle. Thus the $x$-component of $\boldsymbol{F}_\nu$ is given by

$$F_{\nu x}=\int I_\nu\frac{dx}{ds}\,dw \tag{13}$$

where $ds$ is a line element in the direction of $I_\nu$ and $dx$ the $x$-component of $ds$. The expression of $F_{\nu y}$ and $F_{\nu z}$ may be obtained from (13) by substituting $dy$ or $dz$ for $dx$. By using equation (1) it is possible to transform (13) into a remarkable form which affords the most convenient starting-point for any application of the theory to actual problems. We assume the material medium to be isotropic, as will necessarily be the case if we are dealing with gases or liquids. The function $\varkappa_\nu$ must then be a pure function of position, and independent of the direction of emission or absorption.

Writing (1) in the form

$$I_\nu=E_\nu/\varkappa_\nu-\left(\frac{\partial I_\nu}{\partial s}+\frac{1}{c}\frac{\partial I_\nu}{\partial t}\right)\varkappa_\nu^{-1} \tag{14}$$

and introducing this value of $I_\nu$ in (13) and expanding the differentials with respect to $s$, we obtain, finally,

$$\frac{1}{c}\frac{\partial F_{\nu x}}{\partial t}+x_\nu F_{\nu x}=-\frac{\partial P_{xx}}{\partial x}-\frac{\partial P_{xy}}{\partial y}-\frac{\partial P_{xz}}{\partial z}+S_{\nu x} \tag{15}$$

where

$$\left.\begin{aligned} P_{xx}&=\int I_\nu\left(\frac{dx}{ds}\right)^2 dw\ ; \qquad P_{xy}=\int I_\nu \frac{dx}{ds}\frac{dy}{ds}\,dw\ ; \\ P_{xz}&=\int I_\nu \frac{dx}{ds}\frac{dz}{ds}\,dw\ ; \qquad S_{\nu x}=\int E_\nu \frac{dx}{ds}\,dw\ . \end{aligned}\right\} \tag{16}$$

Thus the vector $\boldsymbol{F}_\nu$ satisfies the differential equation

$$\frac{1}{c}\frac{\partial \boldsymbol{F}_\nu}{\partial t}+x_\nu \boldsymbol{F}_\nu+\text{div. } \boldsymbol{P}=\boldsymbol{S}_\nu \tag{17}$$

where $\boldsymbol{P}$ stands for the symmetric tensor of the second rank, the components of which are $P_{xx}$, $P_{xy}$, $P_{xz}$, $P_{yy}$, $P_{yz}$, $P_{zz}$, and which henceforth will be referred to as the "flux tensor."

5. It is of great interest for stellar theory to derive the most general conditions which must be complied with in order that the tensor $\boldsymbol{P}$ shall reduce to an invariant. The principal diagonal components of the tensor are then equal, while the skew components are all zero. Introduce angular co-ordinates of direction $\theta$ and $\phi$ by the expressions

$$\frac{dx}{ds}=\sin\theta\cos\phi\ ; \qquad \frac{dy}{ds}=\sin\theta\sin\phi\ ; \qquad \frac{dz}{ds}=\cos\theta\ .$$

In these co-ordinates the element of solid angle has the form

$$dw=\sin\theta\, d\theta\, d\phi$$

and the equation for one of the skew tensor components, $P_{xy}$ say, thus becomes

$$P_{xy}=\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} I_\nu(\theta,\phi)\sin^3\theta\sin\phi\cos\phi\, d\phi\, d\theta\ .$$

This expression will be zero, provided $I_\nu(\theta,\phi)$ is an even function of $\phi$, and the same will be the case for the other skew components.

For practical applications the dependence of $\boldsymbol{P}$ on $\theta$ is of most importance. Written out in full the expressions of $P_{xx}$, $P_{yy}$, $P_{zz}$ are

$$\left.\begin{aligned} P_{xx} &= \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} I_\nu(\theta,\phi)\,\sin^3\theta\,\cos^2\phi\,d\theta\,d\phi \quad (a)\\ P_{yy} &= \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} I_\nu(\theta,\phi)\,\sin^3\theta\,\sin^2\phi\,d\theta\,d\phi \quad (b)\\ P_{zz} &= \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} I_\nu(\theta,\phi)\,\cos^2\theta\,\sin\theta\,d\theta\,d\phi \quad (c) \end{aligned}\right\} \qquad (18)$$

The equation $P_{xx}=P_{yy}$ is satisfied, provided $I_\nu(\theta,\phi)$ is an even function of $\phi$. The requirement $P_{xx}=P_{zz}$, on the other hand, restricts the dependence of $I_\nu$ on $\theta$. Assume $I_\nu$ to be developed in a power series of $\mu=\cos\theta$:

$$I_\nu=\sum_{n=0}^{\infty}A_n\mu^n\,. \qquad (19)$$

The coefficients $A_n$ in this series will be functions of position and of the angle $\phi$. The condition $P_{xx}=P_{zz}$ now takes the form

$$\sum_{n=0}^{\infty}\int_0^{2\pi}A_n\,d\phi\int_{-1}^{1}\mu^{n+2}\,d\mu=\sum_{n=0}^{\infty}\int_0^{2\pi}A_n\cos^2\phi\,d\phi\int_{-1}^{1}(1-\mu^2)\mu^n\,d\mu\,. \qquad (20)$$

As this condition imposes only a single restriction on an infinite number of coefficients $A_1, A_2 \ldots A_\infty$, it is clear that the desired result can be attained in an infinite number of ways. The case of special interest for practical applications occurs when only the few first terms in the series (19) are appreciable and the coefficients $A_n$ are independent of $\phi$. For the zero term alone, that is, when $I_\nu$ is independent of $\mu$, relation (20) is satisfied identically. This is also the case for the first power term and all terms involving odd powers of $\mu$. The terms involving even powers of $\mu$ will, however, not drop out of formula (20). Thus the form

$$I_\nu(\mu)=A_0+A_1\mu+A_2\mu^2 \qquad (21)$$

cannot be reconciled with the requirement $P_{xx}=P_{zz}$ unless $A_2=0$. As soon as terms beyond $A_3$ are included, the coefficients may be so

adjusted as to comply with the requirement without putting any of them equal to zero. The foregoing result is of interest in so far as it shows that, in the limiting case when the deviations of $I_\nu$ from perfect spherical symmetry are very small, the tensor $\boldsymbol{P}$ deviates very little from an isotropic potential function, and that the tensorial nature becomes apparent as soon as the second-order term in (19) is of importance.

For the case that $A_2$ is zero in (21) we have

$$P_{xx}=P_{yy}=P_{zz}=2\pi\int_{-1}^{+1}(A_0+A_1\mu)\mu^2\,d\mu=\frac{4\pi}{3}A_0\,.$$

The energy density of the radiation field per unit frequency is, on the other hand,

$$A_\nu=\frac{2\pi}{c}\int_{-1}^{+1}(A_0+A_1\mu)d\mu=4\pi\frac{A_0}{c}$$

or

$$P_{xx}=\frac{c}{3}A_\nu\,. \tag{22}$$

Thus, for the limiting case in question, the only surviving components of the flux tensor $\boldsymbol{P}$ are equal to one-third of the energy density of radiation, multiplied by the velocity of light. From expressions (18) it will further be noticed that (22) gives a good approximation in all cases in which the term in $I_\nu$, which is independent of $\theta$ and $\phi$, is large compared to the sum of the terms containing $\theta$ and $\phi$. This circumstance justifies the application of (22) to most problems of transmission of radiation in the interior of a star.[1]

The foregoing considerations all refer to radiation in a specified spectral interval. Integrating $\boldsymbol{F}_\nu$ over all frequencies, we obtain a vector

$$\boldsymbol{F}_r=\int_0^\infty \boldsymbol{F}_\nu\,d\nu \tag{23}$$

which describes the combined flow of radiation for all frequencies.

6. Consider a closed surface $S$ fixed in space. The energy which at any time is contained within $S$ is the sum of three terms: the

[1] *Monthly Notices, R.A.S.*, **84**, 52, 1924.

electromagnetic energy of radiation, the thermal and subatomic energy, and the macroscopic kinetic energy of the substance. The corresponding energy densities we denote by $E_e$, $\rho E$, and $\frac{1}{2}\rho V^2$, $\rho$ and $V$ being mass-density and vectorial velocity of the substance at the point in question. The change in the total energy within $S$ which takes place in the time element $dt$ is thus

$$dt \int \left\{ \frac{\partial E_e}{\partial t} + \frac{\partial}{\partial t}(\rho E) + \frac{\partial}{\partial t}(\tfrac{1}{2}\rho V^2) \right\} d\tau , \tag{24}$$

the integration being extended to the volume inclosed by the surface $S$. This energy must be equal to the sum of the net amount of energy which in the same time interval flows through the surface, and the work performed on the substance within the surface by external forces.

It is convenient to separate from the flux of internal and thermal energy a term $\rho E_s V$ which gives the flux of subatomic energy, the interatomic energy due to rotation of molecules, the excitation of atoms to higher quantum states, etc., $\rho E_s$ being the density of this part of internal energy. The total flux of translational energy of the atoms, on the other hand, is calculated as follows: Denote the velocity of *an individual atom* by $V_i$ and write

$$V_i = V + V_r \tag{25}$$

where $V_r$ is the random velocity of thermal agitation. If we could neglect the effect of collisions between the atoms, the total flux of translational kinetic energy would be given by the expression

$$F_t = \sum_i \left\{ \frac{m_i}{2} V_i^2 V_i \right\} \tag{26}$$

where $m_i$ is the mass of the $i$th atom and where the summation is to be extended to all atoms in unit volume. The collisions we take account of by introducing in (26) a zero-dimensional quantity $P_i$ which is a function of the mean free path of the atoms in question. Expanding (26) by using (25) we obtain

$$F_t = \sum_i \frac{m_i}{2} P_i \{ V^2 V + V_r^2 V + V_r^2 V_r + 2(VV_r)V_r + 2(VV_r)V + V^2 V_r \} . \tag{27}$$

The first term in this expression gives the flux of macroscopic kinetic energy $\frac{1}{2}\rho V^2 \boldsymbol{V}$. The second and third terms give the flux of thermal energy, the second term corresponding to a flux

$$\rho E_t \boldsymbol{V} \; ; \qquad \rho E_t = \sum_i \frac{m_i}{2} P_i V_r^2$$

where $E_t$ is the thermal translational energy of the atoms per unit mass. The third term corresponds to what we are accustomed to call the "conductive flux of energy," which we shall denote by $\boldsymbol{F}_c$. The $x$-component of the fourth term we write in the form

$$V_x \sum_i m_i P_i V_{rx}^2 + V_y \sum_i m_i P_i V_{rx} V_{ry} + V_z \sum_i m_i P_i V_{rz} V_{rx} \qquad (28)$$

which shows that this term is nothing but the inner product of the velocity $\boldsymbol{V}$ with the (symmetric) hydrodynamic stress tensor. It is customary to separate the hydrostatic pressure $p$ from the frictional terms in this tensor, and thus to write its components in the form $p+\pi_{xx}$; $p+\pi_{yy}$; $p+\pi_{zz}$; $\pi_{xy}$; $\pi_{xz}$; $\pi_{yz}$. The last two terms in (27) disappear identically. Thus, finally, the expression for $\boldsymbol{F}_t$ is

$$\boldsymbol{F}_t = \boldsymbol{F}_c + \tfrac{1}{2}\rho V^2 \boldsymbol{V} + \rho E_t \boldsymbol{V} + p\boldsymbol{V} + (\boldsymbol{V}\boldsymbol{\pi}) \; . \qquad (29)$$

If $\boldsymbol{f}$ is the external force per unit mass, there will be an additional increase in energy per unit time and volume of amount $\rho(\boldsymbol{f}\boldsymbol{V})$. Adding up all these terms, we find that expression (24) must be equal to

$$\int \rho(\boldsymbol{f}\boldsymbol{V}) d\tau - \int \{\boldsymbol{F}_r + \boldsymbol{F}_c + (p+\rho E)\boldsymbol{V} + (\boldsymbol{V}\boldsymbol{\pi})\}_n dS \qquad (30)$$

where the first integral is to be extended to the volume within $S$, and the second to the surface $S$ itself, the index $n$ meaning that the components along the outward normal to $dS$ are to be taken for the vectors inclosed in brackets. Transforming the surface integral into a volume integral by Gauss's theorem, and remembering that the surface $S$ is arbitrary and thus may be infinitesimal, we find by combination of (24) and (30)

$$\frac{\partial E_c}{\partial t} + \frac{\partial}{\partial t}(\rho E) + \frac{\partial}{\partial t}(\tfrac{1}{2}\rho V^2) + \operatorname{div}\{\boldsymbol{F}_r + \boldsymbol{F}_c + (\rho E + p)\boldsymbol{V} + (\boldsymbol{V}\boldsymbol{\pi})\} = \rho(\boldsymbol{f}\boldsymbol{V}) \; . \quad (31)$$

This expression may be further simplified by using the hydrodynamical equations

$$\frac{\partial}{\partial t}(\rho \boldsymbol{V}) = \rho \boldsymbol{f} - \nabla p - \operatorname{div} \boldsymbol{\pi} - \operatorname{div} \boldsymbol{M} \; ; \qquad \frac{\partial \rho}{\partial t} + \operatorname{div} \rho \boldsymbol{V} = 0 \tag{32}$$

where the tensor $\boldsymbol{M}$ represents the macroscopic flow of momentum:

$$(\operatorname{div} \boldsymbol{M})_x = \operatorname{div} (\rho V_x \boldsymbol{V}) \; , \quad \text{etc.}$$

Hence

$$\frac{\partial}{\partial t}(\tfrac{1}{2}\rho \boldsymbol{V}^2) = \rho(\boldsymbol{fV}) - (\boldsymbol{V} \nabla p) - (\boldsymbol{V} \operatorname{div} \boldsymbol{\pi}) - \operatorname{div} (\tfrac{1}{2}\rho \boldsymbol{V}^2 \boldsymbol{V}) \; . \tag{33}$$

By using this expression, (31) transforms into

$$\frac{\partial E_e}{\partial t} + \frac{\partial (\rho E)}{\partial t} + \operatorname{div} (\boldsymbol{F}_r + \boldsymbol{F}_c + \rho E \boldsymbol{V}) = (\boldsymbol{V} \operatorname{div} \boldsymbol{\pi}) - \operatorname{div} (\boldsymbol{V\pi}) \; . \tag{34}$$

For viscous fluids $\boldsymbol{\pi}$ has the form

$$\left.\begin{aligned} &\pi_{xx} = \mu \operatorname{div} \boldsymbol{V} - \lambda \frac{\partial V_x}{\partial x} \; ; \quad \pi_{yy} = \mu \operatorname{div} \boldsymbol{V} - \lambda \frac{\partial V_y}{\partial y} \; ; \quad \pi_{zz} = \mu \operatorname{div} \boldsymbol{V} - \lambda \frac{\partial V_z}{\partial z} \\ &\pi_{xy} = -\tfrac{1}{2}\lambda \left( \frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial x} \right) ; \quad \pi_{xz} = -\tfrac{1}{2}\lambda \left( \frac{\partial V_x}{\partial z} + \frac{\partial V_z}{\partial x} \right) ; \quad \pi_{yz} = -\tfrac{1}{2}\lambda \left( \frac{\partial V_y}{\partial z} + \frac{\partial V_z}{\partial y} \right) . \end{aligned}\right\} \tag{35}$$

In these formulae $\lambda$ and $\mu$ are two coefficients of viscosity which, for the case of a gaseous medium, are connected by the relation $\lambda = 3\,\mu$. Carrying out the differentiation processes involved, we find further[1]

$$\Phi = \operatorname{div}(\boldsymbol{V\pi}) - \boldsymbol{V} \operatorname{div} \boldsymbol{\pi} = \pi_{xx} \frac{\partial V_x}{\partial x} + \pi_{yy} \frac{\partial V_y}{\partial y} + \pi_{zz} \frac{\partial V_z}{\partial z}$$

$$+ \pi_{xy} \left( \frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial x} \right) + \pi_{xz} \left( \frac{\partial V_x}{\partial z} + \frac{\partial V_z}{\partial x} \right) + \pi_{yz} \left( \frac{\partial V_y}{\partial z} + \frac{\partial V_z}{\partial y} \right) . \tag{36}$$

[1] Denoting the co-ordinates by $x_1, x_2, x_3$, formula (36) may be written in the form

$$\operatorname{div} (\boldsymbol{V\pi}) - \boldsymbol{V} \operatorname{div} \boldsymbol{\pi} = \sum_{i=1}^{3} \sum_{k=1}^{3} \pi_{ik} \frac{\partial V_i}{\partial x_k} \; . \tag{36a}$$

The right-hand side of (36a) is thus seen to be equal to the *spur* of the tensor product of $\boldsymbol{\pi}$ with the derivative of $\boldsymbol{V}$. (See Eddington's *Mathematical Theory of Relativity*, 1st ed., p. 58.)

With the aid of (35) and (36), equation (34) takes the final form

$$\frac{\partial E_e}{\partial t}+\frac{\partial}{\partial t}(\rho E)+\operatorname{div}\{\boldsymbol{F}_r+\boldsymbol{F}_c+\rho E\boldsymbol{V}\}+p\ \operatorname{div}\boldsymbol{V}=\Phi \qquad (37)$$

where

$$\Phi=2\lambda\left\{\left(\frac{\partial V_x}{\partial x}\right)^2+\left(\frac{\partial V_y}{\partial y}\right)^2+\left(\frac{\partial V_z}{\partial z}\right)^2+\tfrac{1}{2}\left(\frac{\partial V_x}{\partial y}+\frac{\partial V_y}{\partial x}\right)^2+\tfrac{1}{2}\left(\frac{\partial V_x}{\partial z}+\frac{\partial V_z}{\partial x}\right)^2+\tfrac{1}{2}\left(\frac{\partial V_y}{\partial z}+\frac{\partial V_z}{\partial y}\right)^2-\frac{\mu}{\lambda}(\operatorname{div}\boldsymbol{V})^2\right\}. \qquad (38)$$

The quantity $\Phi$ represents the heat energy generated by friction per unit time and volume of the fluid.[1]

Equations (37) and (38) in conjunction with the hydrodynamical equations of motion (32) constitute the general system of differential equations by which the internal state of a star may be described as a function of the time.

If there is no friction in play ($\lambda=0, \mu=0$), and if there is no interchange of energy between radiation and matter ($\partial E_e/\partial t+\operatorname{div}\boldsymbol{F}_r=0$) and no temperature conduction ($\boldsymbol{F}_c=0$), (37) reduces to the adiabatic relation

$$\frac{dE}{dt}+p\frac{d\ 1/\rho}{dt}=0\ ; \qquad \frac{d}{dt}=\frac{\partial}{\partial t}+(\boldsymbol{V}\nabla)\ . \qquad (39)$$

In the case of an ideal gas, $p$ is equal to $\frac{2}{3}\rho E$, and thus equation (39) may be written in the form

$$\frac{d}{dt}(E^3\rho^{-1})=0\ , \qquad (40)$$

showing that $E^3\rho^{-1}$ remains constant for any individual mass element during its motion. If such a relation between $\rho$ and $E$ is fulfilled, not only for an individual mass element, but for all mass elements, the involved constant being the same, the star is said to be in *convective* equilibrium.

Consider, on the other hand, the case in which there is no motion. As the state of the system is thus stationary in time, no term involv-

[1] For an analogous, though less general, treatment of the problem considered in this section, see G. Kirchhoff, *Vorlesungen über Mathematische Physik*, 4 (Leipzig, 1894), 113.

ing the explicit time can appear in the equations. Thus the internal energy density of matter can at most involve a linear term in $t$, and equation (37) reduces to

$$\rho \frac{\partial E}{\partial t} + \operatorname{div} \{F_r + F_c\} = 0 \; ; \qquad \frac{\partial^2 E}{\partial t^2} = 0 \tag{41}$$

which, in conjunction with the hydrostatic equations, is the general condition for the existence of radiative and conductive equilibrium.

Previous investigations concerning the internal constitution of the stars by other authors are, without exception, concerned with purely adiabatic or purely radiative equilibrium. In the sequel we shall consider cases which belong to neither of these classes, and, in particular, study the perturbing effect of local convective atmospheric currents on the flux of radiation from a star, the interior of which is in purely radiative equilibrium. This phase of the problem is likely to have an important bearing on sun-spot phenomena.

7. *Sun-spots.*—We are now prepared to discuss the following hypothesis concerning the origin of sun-spots, in so far as it relates to the reduction in temperature: *The sun as a whole is assumed to be in radiative equilibrium. The spots are regions in which this equilibrium is disturbed by vertical currents.* We shall approach the solution of this problem by successive approximations, beginning with the limiting case of radiative equilibrium.

For the case of radial symmetry the equations of radiative equilibrium may be written in the form

$$\left.\begin{aligned} 4\pi\rho\varepsilon = \frac{1}{r^2}\frac{d}{dr}(r^2 F_r) \; ; \qquad 4\pi\rho G = -\frac{1}{r^2}\frac{d}{dr}(r^2 f) \; ; \qquad -4\pi\varepsilon = \frac{\partial E}{\partial t} \; ; \\ \frac{dP}{dr} = -\rho f \end{aligned}\right\} \tag{42}$$

where $r$ is the distance from the stellar center to the point in question, $F_r$ the radial flux of radiation, $G$ the constant of gravitation, $f$ the gravitational acceleration, and $P$ the sum of gas pressure and radiation pressure. The weak point in all existing theories of stellar constitution lies in the fact that the quantity which we have denoted by $\varepsilon$ is entirely unknown; and we must proceed tentatively by testing different hypotheses concerning the nature of this quantity. The

common practice is to assume $\varepsilon$ constant; in treating problems of stellar atmospheres it is frequently put equal to zero. Such a procedure has, of course, only a heuristic value, but we shall follow it here because the exact value of $\varepsilon$ does not greatly affect the results. For constant $\varepsilon$ it follows from (42) that

$$F_r=\lambda-\frac{\varepsilon}{G}f \tag{43}$$

where $\lambda$ is a constant. As $f$ can vary but little in the stellar atmosphere, this means that $F$ is practically constant throughout the surface layer of the star. This result is not changed by assuming $\varepsilon$ to be zero, and indicates simply that we assume the bulk of the radiation to originate in the interior of the star and not in the surface region under consideration.

The flux vector $\boldsymbol{F}_r$ we assume to be of the form

$$\boldsymbol{F}_r=-\frac{A}{\rho x}\nabla U \qquad \text{or} \qquad F_r=-\frac{A}{\rho x}\frac{\partial U}{\partial r} \tag{44}$$

throughout the whole atmosphere, where $U$ is the energy density of radiation, $A$ a constant, and $x$ the mass absorption coefficient, averaged in a certain way.[1] It follows directly from (22) that (44) is the limiting form for the case of the deep interior of the star, where the asymmetry in the radiant field is small. It is also the limiting form for the flux near the limit of the solar surface, where the radiation is approximately tubular, as considered in the Schuster-Schwarzschild approximation; but the constant $A$ will be several times larger in the latter case than in the former. Assuming $x$ to be a constant in (38), eliminating $dr$ from the hydrostatic equation in (42) by using (44), and integrating, we obtain

$$P=\frac{fA}{F_r x}(U-U_0)+\beta U_0\,; \qquad 1>\beta>\tfrac{1}{3} \tag{45}$$

where $U_0$ is the energy density of radiation corresponding to the temperature of matter at the top of the atmosphere. Hence, by Stefan's law, $P$ will be sensibly proportional to the fourth power of the temperature $T$ as soon as we are so far down in the atmosphere

[1] *Monthly Notices, R.A.S.*, **84**, 525, 1924.

that $U_0$ is small compared to $U$. This result depends little upon the assumed constancy of $x$. Using, for instance, the formula

$$x = \text{const. } \rho T^{-3}, \tag{46}$$

which may give a better approximation in consideration of the conditions in the star's interior,[1] we find, similarly, radiation pressure being neglected,

$$P = \text{const. } U \sqrt{1 - U_0^2/U^2} \tag{47}$$

which, for the case of the stellar photosphere, does not differ essentially from (45). Hence, for an atmosphere in radiative equilibrium, we assume that the gas pressure falls off as the fourth power of temperature. The mass density will consequently be proportional to the temperature in the third power. This result will be used in what follows.

As a next step we consider the case of an atmosphere in radiative equilibrium except for isolated places where local vortices and convection currents disturb the equilibrium conditions. We neglect the effect of friction and temperature conduction, put $\varepsilon$ equal to zero, assume the state to be stationary in time, and write (37) in the consequent form

$$\operatorname{div} \boldsymbol{F}_r = \left\{ V\left( \nabla P - \rho \nabla E - \rho \nabla \frac{P}{\rho} \right) \right\}. \tag{48}$$

For an ideal gas mixture we have

$$P = \tfrac{2}{3} \rho E$$

and consequently

$$\nabla P - \rho \nabla E - \rho \nabla \frac{P}{\rho} = \tfrac{2}{3} E \nabla \rho - \rho \nabla E. \tag{49}$$

The vectors $\nabla \rho$ and $\nabla E$ may point in slightly different directions, but it lies in the nature of our assumptions that this difference will be small, and we assume that a good approximation will be obtained by taking $\rho$ a function of $E$. Thus for $\rho$ proportional to $E^n$, expression (49) assumes the form

$$\frac{n - \frac{3}{2}}{n+1} \nabla P. \tag{50}$$

[1] *Mt. Wilson Contr.*, No. 296; *Astrophysical Journal*, 61, 267, 1925.

If $n$ is equal to $\frac{3}{2}$, this expression is zero, and we have the case of convective equilibrium (cf. equation 40). The value of $n$ will, however, be considerably larger than $\frac{3}{2}$, since that corresponding to radiative equilibrium is of the order of 3. Thus

$$a=\frac{n-\frac{3}{2}}{n+1} \tag{51}$$

is a positive quantity between zero and $\frac{3}{8}$. For small values of the vertical acceleration, the hydrostatic equation in (42) will give a good approximation to the pressure gradient, and thus (48) may be brought into the final form

$$\text{div } \boldsymbol{F}_r = a\rho(\boldsymbol{V}f) \ . \tag{52}$$

This equation expresses the fact that, apart from the multiplier $a$, the increase in intensity of the flux of radiation is balanced by the work done by the gravitational forces on the streaming masses.

To obtain the total loss in intensity of radiation caused by local stationary convection current, we integrate (52) throughout a tube which runs parallel to the direction of $\boldsymbol{F}$ from the base of the current to the top of the stellar atmosphere. Let $\sigma_1$, $\sigma_2$ be the lower and upper cross-sections of this tube, and $F_1$, $F_2$ the mean values of $F_r$ over $\sigma_1$ and $\sigma_2$, respectively. For the case under consideration the difference between $\sigma_1$ and $\sigma_2$ will be neglected, and the tube assumed to be straight, and of length $H$. The volume of the tube is thus $\sigma H$. Transforming the volume integral over the div-expression in (52) into a surface integral by Gauss's theorem, and denoting mean values over the volume $\sigma H$ by a horizontal bar, we find

$$F_2-F_1=\overline{aHV_z f\rho} \tag{53}$$

where $V_z$ denotes the vertical component of the velocity. A rigorous test of this formula is not possible, because of our ignorance of most of the quantities which enter; but, on the assumption that (53) is true, it may be found useful in various ways. We know, for instance, with fair certainty that in the case of sun-spots the value of $\bar{\rho}$ cannot be much larger than $10^{-6}$ g/cm$^3$ and that $V_z$ cannot much exceed 10 km/sec. The temperature of the spots is frequently found

to be about 1000° K below the average temperature of the solar surface, which is of the order 6000° K. Hence, by Stefan's law,

$$F_1 - F_2 = \tfrac{1}{2}F_1 = 4 \cdot 10^{10} \text{ c.g.s.}$$

Using these data and the limiting value $\frac{3}{8}$ for $a$, we find from (47) that the thickness of a sun-spot cannot be less than fifty kilometers. This value is very small compared to the horizontal extension of spots, and it is, therefore, possible to assume very much smaller values of $\bar{\rho}$ and $V_z$ without arriving at improbably large values of the vertical thickness.

A more refined test of the theory is scarcely to be obtained, even with considerably improved observational material; and we must, provisionally, be content with the result that sun-spots may be the natural results of solar photospheric convection currents of moderate strength, even though observations do not provide a definite proof of this conclusion.[1]

7. *Solar rotation.*—It is a significant fact that the sun does not rotate like a rigid body for which the instantaneous angular velocity is the same for all mass elements. In the reversing layer of the sun the angular velocity $W$ follows the law[2]

$$W = W_0 + W_1 \cos^2 \phi \tag{54}$$

where $\phi$ is the solar latitude; $W_0$ and $W_1$ are constants. It is naturally to be expected that the stars in general will exhibit similar deviations from a state of uniform rotation, which we shall call the *general circulation* of the star, in contradistinction to regions of local circulation such as sun-spots.

[1] Bjerknes (*Comptes Rendus*, **182**, 48, 1926) has suggested that the depression of the isothermal surfaces in a sun-spot may be connected with a special kind of horizontal circulation [cf. formula (66) of this paper]. Although the assumptions underlying the quantitative estimates in Bjerknes' paper may differ widely from actual conditions, it seems that a structure of the horizontal velocity field of a sun-spot, essentially of the kind contemplated by Bjerknes, may just be necessary in order to produce the vertical motion contemplated in the foregoing theory, and the two theories may, therefore, be consistent in the end. For a further development of this theory, see also Bjerknes, *Mt. Wilson Contr.*, No. 312; *Astrophysical Journal*, **64**, 1926.

[2] W. S. Adams, "An Investigation of the Rotation Period of the Sun," *Carnegie Institution Publications*, No. 138, p. 130.

The origin of the general circulation of the sun is probably similar in nature to that of the terrestrial atmosphere. The terrestrial circulation is maintained by the systematically higher temperature of the equatorial air masses relative to the polar masses, which disturbs the conditions of hydrostatic equilibrium and starts the complicated system of air currents between equatorial and polar regions. Although the general circulation of the terrestrial atmosphere arises from non-uniform heating *from without*, that of the sun may, on the other hand, be caused by non-uniform heating *from within*.

In order to see how a permanent circulation may thus be started, consider the case of a star having a steady motion such that the acceleration of an individual mass element arises from a potential $\Omega$. This implies the existence of a velocity potential $\omega$ which is connected with $\Omega$ by the relation[1]

$$\Omega=\frac{\partial\omega}{\partial t}+\tfrac{1}{2}(\nabla\omega)^2\ ; \qquad V=\nabla\omega\ ; \qquad \frac{dV}{\partial t}=\nabla\Omega\ .$$

Denoting gas *plus* radiation pressure by $P$, and the gravitational potential by $\phi$, we obtain the equations of motion in the form

$$\frac{1}{\rho}\nabla P=\nabla(\phi-\Omega)\ ; \qquad -\nabla^2\omega=\frac{d\rho}{\rho dt}\ , \tag{55}$$

frictional terms being neglected. Operating by curl on both sides of this equation, we find that the vector product of $\nabla\rho$ and $\nabla P$ is zero, and hence that these two vectors are parallel, and, by (55), parallel to $\nabla(\phi-\Omega)$. The surfaces of constant density and pressure will therefore coincide with the surfaces,

$$\phi-\Omega=\text{const.}\ ,$$

which we shall call "level surfaces." As the temperature is a function of density and pressure, it follows that temperature will also be constant over a level surface.

The thermal structure of the star is essentially determined by the distribution of energy-generating sources, as specified by the function $\varepsilon$ [cf. equation (41)]. Except for a transition layer somewhere

[1] See H. Lamb, *Hydrodynamics*, 5th ed. (Cambridge, 1924), art. 20.

in the stellar atmosphere, the flux vector will have the form (44). Since $U$ is a function of temperature only, the normal surfaces of $\boldsymbol{F}_r$ are level surfaces. Denoting $\phi - \Omega$ by $\psi$, we may thus write

$$\boldsymbol{F}_r = -A\nabla\psi \tag{56}$$

where $A$ is some function of $\psi$, and, by (37) and (41),

$$4\pi\varepsilon\rho = \operatorname{div} \boldsymbol{F}_r - P \operatorname{div} \boldsymbol{V} = -A\nabla^2\psi - \frac{\partial A}{\partial \psi}(\nabla\psi)^2 - P\nabla^2\omega \tag{57}$$

where, by Poisson's equation,

$$\nabla^2\psi = -4\pi G\rho - \nabla^2\Omega\,. \tag{58}$$

Although it is not known just what law $\varepsilon$ will follow, it is a fair guess that for regions in the same star having the same temperature and density $\varepsilon$ will also be the same. Thus $\varepsilon$ should be constant over a level surface: and hence, by (57),

$$A\nabla^2\psi + \frac{\partial A}{\partial \psi}(\nabla\psi)^2 + P\nabla^2\omega = \text{function of } \psi\,. \tag{59}$$

In case of rigid rotation with constant angular velocity $W$

$$\nabla^2\Omega = -2W^2\,; \qquad \nabla^2\omega = 0\,.$$

Since $(\nabla\psi)^2$ varies over a level surface, it is, by (59), necessary that $A$ be independent of $\psi$; and hence (57) assumes the form

$$\varepsilon = \text{const.}\left(1 - \frac{W^2}{2\pi G\rho}\right), \tag{60}$$

a relation first derived by von Zeipel.[1] If the internal heating system of the star does not conform to this particular law, rigid rotation becomes impossible, and a more general kind of motion will take its place. It seems physically impossible, however, as pointed out by Eddington,[2] that this law should be followed near the stellar surface, since $\varepsilon$ becomes negative in the region where $\rho < W^2/2\pi G$. Near the surface of a rotating star we must, therefore, always expect some kind of general circulation.

The result embodied in equation (60) is evidently of a more re-

[1] *Monthly Notices, R.A.S.*, **84**, 665, 1924.

[2] *The Observatory*, **48**, 73, 1925.

strictive character than the analogous criterion of stability established by Poincaré,[1] which we shall recall briefly. If a star rotating with constant angular velocity $W$ is to remain in permanent equilibrium, it is necessary that the apparent force be directed inward all over the surface $S$ which forms the free surface of the star. Thus we have the condition that the integral of $\partial\psi/\partial n$ over the surface $S$ must be negative, $n$ denoting the outward normal to $S$. Using Gauss's theorem and equation (58), we thus have the relation

$$\int_S \frac{\partial\psi}{\partial n}\, dS = -\int_{\text{vol. } S} (4\pi G\rho + \nabla^2\Omega) d\tau\,. \tag{61}$$

Let $\bar{\rho}$ denote the mean density of the star, defined as total mass divided by total volume, which latter may be denoted by $T$. For rigid rotation $\nabla^2\Omega = -2W^2$, and the foregoing equation takes the form

$$\int_S \frac{\partial\psi}{\partial n}\, dS = -2T(2\pi G\bar{\rho} - W^2)\,. \tag{62}$$

For equilibrium to be possible the right-hand expression must be negative, and therefore

$$\frac{W^2}{2\pi G\bar{\rho}} < 1 \qquad \text{or} \qquad P > \frac{3.1}{\sqrt{\bar{\rho}}} \text{ days} \tag{63}$$

where $P$ is the period of rotation, and $\rho$ is measured with the density of air as unity.

It seems to be implicitly assumed by all investigators of the theory of rotating stars that if a star, originally rotating with constant angular velocity, enters a state in which relation (63) is violated, cataclysmic motion must occur, resulting in fission or formation of a sharp edge from which matter is thrown off continuously. These conclusions, however, rest upon the hypothesis that the stars will always retain a state in which the angular velocity is the same for all mass elements. The impossibility of this hypothesis is clearly shown, however, by von Zeipel's relation (60); and in the sequel we shall examine some simple cases and show how a star may take up a state of motion with varying angular velocity, so as to conform to particular laws for the generation of stellar energy.

[1] *Figures d'Équilibre d'une Masse Fluide* (Paris, 1902), p. 11.

In examining the characteristics of the resulting motion, we shall first show that it is not sufficient to assume the angular velocity to be a function of only the distance $R$ from the axis of rotation. For in this case an acceleration potential exists:

$$\Omega = -\int W^2(R)RdR\ ; \qquad \nabla^2\Omega = -2W^2 - R\frac{\partial W^2}{\partial R}\ ; \qquad \nabla^2\omega = 0 \quad (64)$$

and equation (59) must be satisfied. Even without detailed calculations, it is seen that the form (64) of $\Omega$ is, in general, incompatible with (59), since $\Omega$ will be constant over a certain class of cylindrical surfaces, while $(\nabla\psi)^2$ will be constant over certain oblate spheroidal surfaces, not very different from the surfaces of constant $\psi$.

In the next step we give up the assumption that an acceleration potential exists, but still assume that the motion takes place in circles around a fixed axis, such that the angular velocity $W$ is now also a function of the distance $z$ from the equatorial plane. Denoting by $\boldsymbol{r}$ the radius vector from the center of the star to the point in question the hydrodynamic equation of motion takes the form

$$[\boldsymbol{WV}] = \nabla\phi - \frac{1}{\rho}\nabla P\ ; \qquad \boldsymbol{V} = [\boldsymbol{Wr}]\ , \qquad (65)$$

viscosity being neglected. Operating by curl on both sides of (62), we find, after some calculations,[1]

$$2\boldsymbol{V}\frac{\partial W}{\partial z} = [\nabla\alpha\nabla P]\ ; \qquad \alpha = \frac{1}{\rho}\ . \qquad (66)$$

This relation shows that when the angular velocity varies in a direction parallel to the axis of rotation, the isobaric and isosteric

[1] By ordinary vector formulae: curl $[\boldsymbol{WV}] = \boldsymbol{V}$ div $\boldsymbol{W} + (\boldsymbol{W}\nabla)\boldsymbol{V} - (\boldsymbol{V}\nabla)\boldsymbol{W} - \boldsymbol{W}$ div $\boldsymbol{V}$. Since div $\boldsymbol{V} = 0$ and $\boldsymbol{W}$ remains unaltered for an individual particle during the motion the last two terms disappear. Remembering that $\boldsymbol{W}$ has only a $z$-component, the first two terms may be written in the form $\frac{\partial}{\partial z}(W_z\boldsymbol{V})$, and, by the fact that $\boldsymbol{V}$ is proportional to $W_z$ and otherwise independent of $z$, it follows that

$$\frac{\partial}{\partial z}(W_z\boldsymbol{V}) = 2\frac{\partial W_z}{\partial z}\boldsymbol{V}\ .$$

Hence the left side of (66). The right side follows from the formula curl $\mu\boldsymbol{A} = \mu$ curl $\boldsymbol{A} + [\nabla\mu\boldsymbol{A}]$ by putting $\mu = \alpha$ and $\boldsymbol{A} = \nabla P$, and remembering that curl $\nabla P$ is identically zero.

surfaces will be inclined to each other by a finite angle. The flux of radiation, since temperature is a function of both density and pressure, will be parallel neither to the pressure nor to the density gradient. The variation of density along an isobaric surface is directly determined by (63). If $\boldsymbol{W}$ is directed along the positive $z$-axis, as we have assumed, the vectors $\nabla P$ and $\nabla a$ will lie in planes through this axis, and $\boldsymbol{V}$ will be directed parallel to the positive $y$-axis.

Consider conditions in the upper right quadrant of the $xz$-plane. With the foregoing conventions the positive direction of the angle $\varphi$ between the vectors $\nabla a$ and $\nabla P$ is to be counted in the direction from the positive $z$ to the positive $x$-axis. If $\partial W/\partial z$ is positive, the right-hand side of (63) must also be positive, meaning that $\varphi$ is less than $\pi$ which again means that at that point the isobaric surfaces ($P=$ const.) are *more* oblate than the isosteric surfaces ($a=$const.). If $\partial W/\partial z$ is positive all the way from pole to equator over a particular isobaric surface, this surface will consequently be coldest at the pole and hottest at the equator. If $\partial W/\partial z$ is negative, conditions are exactly reversed.[1]

We have thus seen that by suitably varying the angular velocity parallel to the axis of rotation, the isobaric surfaces can be tilted with respect to the isosteric and isothermic surfaces to any desired angle, so that any particular law of generation of energy can be complied with. A certain amount of convection is, however, necessary in all cases, because of internal friction. Let us assume for a moment that the angular velocity is adjusted to the correct tilt of the surfaces. The effect of viscosity will always be to reduce this tilt, and a reinforcement of the motion can be obtained only by introducing a current system from pole to equator, or vice versa, which by the principle of conservation of angular momentum, will increase or decrease the angular velocity by any required amount. From the previous calculations concerning the temperature of sun-spots it may be concluded, however, that any large-scale, vertical convection currents possibly existing in the sun must be of an exceedingly feeble intensity in order not to disturb the uniform illumination of the solar surface. It seems doubtful, therefore, that such currents can, for the case of

[1] These results were originally found by Bjerknes, "Dynamics of the Circular Vortex," *Geofysiske Publikationer*, 2, No. 4, 1921, p. 44 (*A*), p. 45 (*B*).

the sun, materially affect the distribution of different elements in the sun's interior, as has been suggested.[1]

In order to illustrate the conditions more clearly, consider a simple example. Assume the liberation of energy to be proportional to density, and the flux vector to have the form

$$F_r = -\nabla U$$

where $U$ is a function of the absolute temperature $T$ only. We have then the following system of equations:

$$\left.\begin{aligned} -W^2R &= \frac{\partial\phi}{\partial R} - \frac{1}{\rho}\frac{\partial P}{\partial R} \quad (a) \quad \nabla^2 U = -4\pi\varepsilon\rho \quad (c) \quad P = k\rho T \quad (e) \quad k = \text{const.} \\ 0 &= \frac{\partial\phi}{\partial R} - \frac{1}{\rho}\frac{\partial P}{\partial z} \quad (b) \quad \nabla^2\phi = -4\pi G\rho \quad (d) \quad \varepsilon = \text{const.}\,, \end{aligned}\right\} \tag{67}$$

frictional terms as well as radiation pressure being neglected. By using $(e)$, equations $(a)$ and $(b)$ may be written in the form

$$W^2 = \frac{1}{R}\frac{\partial}{\partial R}(kT-\phi) + k\frac{T}{R}\frac{\partial \log\rho}{\partial R} \tag{68}$$

$$\frac{\partial \log\rho}{\partial z} = -\frac{1}{kT}\frac{\partial}{\partial z}(kT-\phi)\,. \tag{69}$$

These equations show that for the entire class of solutions of $(c)$ and $(d)$, in which the boundary conditions for $U$ and $\phi$ are the same, in consequence of which $T$ is a function of $\phi$, the density is given by an expression of the form

$$\log\rho = -\int\frac{d(kT-\phi)}{kT} + \text{function of } R\,. \tag{70}$$

By introducing expression (70) in (68) it follows that the angular velocity is given by the expression

$$W^2 = kTA(R) \tag{71}$$

where $A$ is an arbitrary positive function of $R$ only.

Thus, passing along a line parallel to the axis of rotation, $W$ increases from the surface to the interior, being proportional to the

[1] A. S. Eddington, *The Observatory*, 48, 73, 1925.

square root of the temperature, and the star will, by (66), be hottest at the poles and coldest at the equator. For cases which are more general than those considered above, the function $A$ will probably be completely determined except for an arbitrary constant. If this should prove to be true, observations of the rotation of the sun might provide a direct source of information about the origin of the energy of solar radiation.

MOUNT WILSON OBSERVATORY
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# ORBIT OF THE SPECTROSCOPIC BINARY 19 $\tau^5$ ERIDANI

By OTTO STRUVE

ABSTRACT

The orbital elements of this spectroscopic binary with double lines were derived from forty spectrograms obtained with the one-prism Bruce spectrograph of the Yerkes Observatory. $P = 6^d.2236$; $\gamma = +15.0$ km/sec.; $e = 0.20$; $K_1 = 107$ km/sec.; $K_2 = 103$ km/sec.; $\omega = 313°$; $T = 1925$ Oct. 23.05 G.C.T.; $a_1 \sin i = 8{,}640{,}000$ km; $a_2 \sin i = 8{,}970{,}000$ km; $m_2 \sin^3 i = 2.87\ \odot$; $m_1 \sin^3 i = 2.76\ \odot$; $m_1/m_2 = 0.96$.

This is one of the nine stars designated, according to Bayer, by the Greek letter $\tau$. Its position for 1900 is $\alpha = 3^h29^m.4$, $\delta = -21°58'$. Its visual magnitude, according to the *Henry Draper Catalogue*, is 4.32; its photographic magnitude is 4.27; and its spectral type is B8. The spectral lines are broad and diffuse and would justify the designation of spectrum as B8n. The hydrogen lines are strong and very broad and are unsuitable for measurement. Those of helium are faint, as would be expected in a star of class B8. The same is true of the silicon lines, which are frequently strong in this type. The only line suitable for measurement is that of ionized magnesium at $\lambda$ 4481.400, but even this line is faint and ill defined.

The binary nature of $\tau^5$ Eridani was discovered by Professor Edwin B. Frost when he observed the duplicity of the lines.[1] The first plate obtained by him in December, 1905, showed single lines, while on the next plate, obtained in October, 1906, a separation of nearly 200 km/sec. was indicated. The two components have nearly the same intensity.

The observations of this star were resumed in 1924 and 1925, and forty plates were obtained for the determination of the orbit. Two of the early plates had to be omitted on account of the indistinctness of their lines. Table I contains the results of the measurements. I am indebted to Professor S. B. Barrett for part of the exposures. All of the spectrograms were obtained with the Bruce spectrograph, with a dispersion of one prism and with a camera of 24 inches focal length. For all the recent plates the Ross camera lens was used.

[1] *Astrophysical Journal*, 25, 63, 1907.

Greenwich Civil Time (Universal Time) is given for all observations after January 1, 1925.

TABLE I

| Date G.M.T. | Observer | Quality | Velocity I km/sec. | Velocity II |
|---|---|---|---|---|
| 1905 Dec. 15.615 | F, S | p | − 5.5 | ..... |
| 1906 Oct. 19.758 | F, B, S | g | − 96.3 | + 84.8 |
| Nov. 1.747 | P, S | g | − 60.6 | + 86.2 |
| Dec. 14.688 | Fox, S | vp | + 40. | ..... |
| 1908 Jan. 28.533 | F, B | g | + 11.1 | ..... |
| 1924 Dec. 14.667 | σ, S | g | − 3.6 | ..... |
| Dec. 27.690 | B, S | f | − 5.7 | ..... |
| 1925 Jan. 6.113 | σ, S | g | + 14.7 | ..... |
| Jan. 8.064 | B, S | g | −115.2 | + 91.9 |
| Jan. 11.045 | σ | f | − 69.9 | + 60.1 |
| Jan. 15.063 | σ | f | − 65.2 | + 38.3 |
| 1925 Jan. 18.067 | σ, S | f | − 2.0 | ..... |
| Jan. 21.051 | σ | g | −101.1 | + 88.8 |
| Jan. 22.999 | σ | g | − 96.2 | + 99.5 |
| Jan. 23.103 | σ, S | p | − 56.4 | + 80.7 |
| Jan. 31.001 | σ | p | + 43.7 | ..... |
| Feb. 1.003 | B | p | − 72.1 | + 93.0 |
| Feb. 5.007 | B | f | −113.7 | + 94.6 |
| Feb. 12.003 | B | g | + 43.2 | ..... |
| Feb. 15.015 | B | g | − 40.5 | + 76.6 |
| Mar. 2.029 | σ | p | − 40.2 | +135.4 |
| Aug. 31.427 | σ, S | g | − 69.1 | +129.8 |
| Sept. 3.431 | σ, S | g | − 90.4 | +123.2 |
| Sept. 21.399 | σ | g | + 31.9 | ..... |
| Sept. 24.390 | B, S | g | + 2.0 | ..... |
| Sept. 28.350 | B, S | g | − 82.0 | +107.4 |
| Oct. 5.381 | σ, S | g | − 90.7 | +130.9 |
| Oct. 15.333 | B, S | g | − 58.4 | + 99.1 |
| Oct. 18.367 | σ, S | g | − 65.4 | + 46.3 |
| Oct. 22.348 | σ, S | g | + 26.5 | ..... |
| Oct. 29.283 | σ, B, S | g | − 97.7 | +107.5 |
| Nov. 1.265 | B, S | g | − 80.2 | +125.0 |
| Nov. 2.287 | B, σ, S | g | −117.7 | +109.9 |
| Nov. 9.325 | σ, S | g | − 42.5 | + 31.0 |
| 1925 Nov. 14.226 | σ, S | p | − 80.0 | + 90.0 |
| Nov. 18.256 | σ, S | g | − 87.7 | + 93.9 |
| Nov. 19.187 | B, S | g | + 20.1 | ..... |
| Nov. 26.226 | σ, S | g | − 84.1 | +127.8 |

The names of the observers and the quality of the plates are designated as follows: B=S. B. Barrett; F=E. B. Frost; P=J. A. Parkhurst; Fox=P. Fox; S=F. R. Sullivan; σ=O. Struve; f=fair; g=good; p=poor; v=very.

After a number of trials the period was found to be $6^d.2236$. This value represents also the old observations of 1905–1908. However, the number and quality of these observations is insufficient to guarantee the last two decimals of the period. I have been unable to find any other published velocities of this star. Dr. J. H. Moore, of the Lick Observatory, has kindly advised me that the star has been

observed at Mount Hamilton with a dispersion of three prisms. The magnesium line 4481 appeared double on a spectrogram taken September 7, 1906, and possibly also on January 13, 1904. Dr. Moore concludes: "The spectrum is too poor for three-prism dispersion and should be obtained with a one-prism spectrograph."

The orbit was determined from the velocity curve by means of the graphical methods of Lehmann-Filhés and of K. Laves. In the

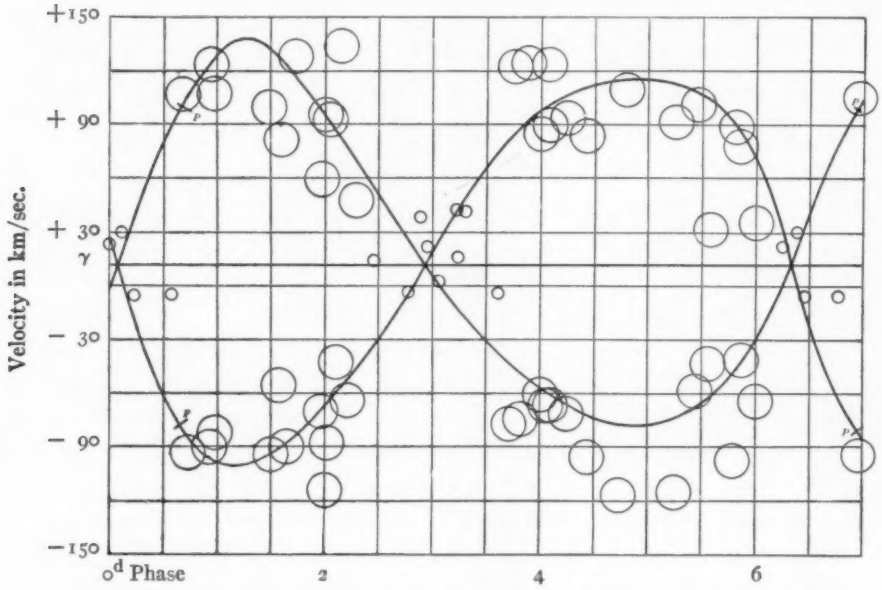


FIG. 1.—The large circles represent the individual velocities for both components; the small circles give velocities derived from blended lines.

latter case Schwarzschild's method was used for determining $T$. The mean of these two independent sets of elements gives a satisfactory representation of the observations.

ORBITAL ELEMENTS OF 19 $\tau^5$ ERIDANI

| | |
|---|---|
| $\gamma$ = 15.0 km/sec. | $a_1 \sin i$ = 8,970,000 km |
| $P$ = $6^d.2236$ | $a_2 \sin i$ = 8,640,000 km |
| $e$ = 0.20 | $m_2 \sin^3 i = 2.87 \odot$ |
| $K_1$ = 107 km/sec. | $m_1 \sin^3 i = 2.76 \odot$ |
| $K_2$ = 103 km/sec. | Mass ratio = 0.96 |
| $\omega$ = 313° | Residual velocity = −1.5 km/sec. |
| $T$ = Phase $0^d.70$ = 1925 Oct. 23.05 G.C.T. | |

It will be seen from the figure that the residuals (O.−C.) are large. This is undoubtedly due to the uncertainty of the measurements. In view of the poor character of the spectrum no least-squares solution was made.

In the figure, phase 0.000 equals 1925 Oct. 22, $8^h21^m$ G.C.T. = J.D. 2424446.348.

YERKES OBSERVATORY
December 1, 1925

# THE DOUBLE STAR 9 ARGUS

By S. A. MITCHELL

The interesting character of the visual binary $\beta$ 101 ($7^h47^m.1$, $-13°38'$, 1900.0) has been mentioned in this *Journal* as follows: In **58,** 141, 1923, Otto Struve discusses the spectroscopic observations made at the Yerkes Observatory in connection with Aitken's orbit. He finds the most probable value of the parallax to be $+0''.078$. In **60,** 201, 1924, the writer gives the trigonometric parallax for the second series of plates taken at the McCormick Observatory. In this article it was stated that a third series of plates was being taken. These plates have now been measured. The values from each of the three series of plates are as follows:

| Series | Relative Parallax | | Dates of Series | No. of Plates | Seasons |
|---|---|---|---|---|---|
| I..... | $+0''.036$ | $\pm 0''.008$ | 1915.2–1917.9 | 22 | 6 |
| II..... | .028 | .008 | 1920.8–1923.3 | 23 | 6 |
| III..... | .027 | .009 | 1923.8–1925.9 | 20 | 5 |
| Mean.. | +0.030 | ±0.005 | ..... | ..... | ..... |

Three comparison stars were utilized for each of the first two series of plates. Five comparison stars were used in the third series, and the plates were taken under good conditions of seeing. All plates were measured by the writer. There is a remarkably good accord between the values from the three series.

The McCormick value differs greatly in size from that of the Sproul Observatory, the latter value being $+0''.121 \pm 0''.009$, derived by photography. Flint's value from meridian circle is $+0''.035 \pm 0''.026$, and the spectroscopic value of Mount Wilson is $+0''.079$.

The absolute value of the McCormick parallax is $+0''.034$, from which a mass of the system 15.2 $\odot$ is derived. According to Struve, the brighter component has a spectral class F8. Assuming the relative masses of the components as 10:4 (*Publications McCormick Observatory*, **3,** 237, 1920), then we derive the masses 10.8 $\odot$ and

4.4 ⊙ for the two components, which are larger masses than we are led to expect. On account of the splendid accord between the values of the parallax from the three McCormick series it does not seem advisable to take a fourth series of plates.

In this *Journal*, **60**, 201, 1924, the writer called attention to the brilliant star Arcturus, for which the spectroscopic parallax determined by four observatories, Mount Wilson, Lockyer, Harvard, and Victoria, gives a far higher value than the trigonometric parallax from photographic measures at Allegheny, Yerkes, and McCormick observatories. The star 9 Argus affords another well-authenticated case of the dependence of the spectroscopic parallax on the mass of the star.

LEANDER MCCORMICK OBSERVATORY
UNIVERSITY OF VIRGINIA
February 27, 1926

# *REVIEWS*

*The Development of the Sciences.* Edited by L. L. WOODRUFF. New Haven: Yale University Press, 1923. 8vo. Pp. xiv+327. Plates 16. $3.50.

This book consists in the main of a series of public lectures delivered by some of the professors of Yale University during the year 1922, and particularly intended for the students of science.

The lectures, six in number, cover from thirty to fifty pages each, and deal with the historical development of the major sciences. Together they form a brief history of science. The topics and authors are as follows: mathematics, E. W. Brown; physics, H. A. Bumstead; chemistry, John Johnston; astronomy, Frank Schlesinger; geology, H. E. Gregory; biology, L. L. Woodruff, who is also editor of the volume.

This book is a very welcome addition to those on the history of the sciences of which there are altogether too few in English. The lecturers have had a difficult task to cover in such brief space even the main points of man's progress in science. The teacher interested in astronomy will probably regret that more space was not devoted to the history of the development of the telescope and spectroscope which have reached such a high point of efficiency in astronomical research at the present time. No mention is made of the contribution of Newton or William Herschel to the reflecting telescope or of the later development of the refracting and reflecting telescopes.

On the whole one finds very little overlapping and very few omissions. The monographs are of distinct literary quality and the scholarly character of the work of the contributors is an honor to the university they represent. It is to be hoped that a companion series of lectures may take up the development of the sciences where these have left off, giving the student and general reader a description of the remarkable progress that has been made during the past decade and the big problems that are now in the process of investigation and solution.

Mention should be made, in this connection, of the enrichment of the volume under review by ninety-four portraits of eminent men of science; and the Appendix in which are given three hundred brief biographies of

the more important scientists; a Bibliography of one hundred and fifty books for reference and collateral reading; a table of chemical elements arranged in the chronological order of their discovery, and the terms of the geologic column, their order and significance.

W. H. GARRETT

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*Œuvres Complètes de Christiaan Huygens*. Vol. XV. Publiées par la SOCIÉTÉ HOLLANDAISE DES SCIENCES. La Haye: M. NIJHOFF, 1925. Pp. 618. Price, $6.00.

The fifteenth volume of the complete works of Christiaan Huygens, published by the *Hollandsche Maatschappij der Wetenschappen* in Haarlem brings us, in a splendidly executed form, the astronomical work of Huygens.

The compilation of Huygens' contributions to astronomy, which, outside of a few printed articles, had to be derived from a series of manuscripts, letters, and loose leaves, was no simple matter, but the editors of the present volume, Professor D. J. Korteweg, of Amsterdam, and Professor A. A. Nijland, of Utrecht, have succeeded in giving us a careful review of Huygen's astronomical work from 1655 to 1694.

The Introduction discusses in detail the sources and the relative value of the different available manuscripts. The oldest observations, prior to 1657, seem to have been collected in the *parvum libellum*, which, however, has been lost. Happily, some of these observations were used by Huygens in his *Systema Saturnium*, but some others, as probably his observations on P Cygni, are supposedly lost.

A very complete account is given of the lenses made by Christiaan Huygens and his brother Constantyn; it seems that the latter even had the major part in this work, which is the more remarkable as he was no scientist, but a poet and a diplomat.

After the Introduction the observations follow in chronological order, illustrated with many very interesting sketches. Then come the three principal papers by Huygens: "De Saturni Lunâ Observatio Nova" (1656); "Systema Saturnium" (1659); and his "Brevis Assertio Systematis Saturnii" (1660), which is an answer to the "Brevis Annotatio in Systema Saturnium," by Eustachio Divini, which had appeared the year before. All these articles are printed in French and Latin in the present volume.

Up to 1655 only the four moons of Jupiter discovered by Galileo were known, and although Huygens, with his better telescopes, scrutinized

Venus, Mars, and Saturn, he could add only Titan to the list of satellites. He gave as the synodic period about 16 days, while in reality it is $15^{d}\ 23^{h}\ 15^{m}$.

A good example of Huygens' clear and logical way of reasoning is given in the *Systema Saturnium*. He bases his hypothesis to explain the baffling appearance of Saturn in its different phases on what the observations of Titan had taught him. Titan's period being 16 days, Huygens concludes, in analogy with other phenomena in the solar system, that the rotation of Saturn must be performed in considerably less time, and that the appendices must revolve around the planet in a time slightly longer than the rotation. Since these appendices have the same appearance from day to day for a considerable time, it must be that the planet is surrounded by a symmetrical body, which cannot be anything but a ring, detached from the main body and having an axis of rotation at right angles to its plane. Also, as the phases of the ring are quite different at various times, the plane of the ring must form an angle with the ecliptic, which angle he gives as 20°. Huygens had already published this hypothesis in the anagram at the end of his "De Saturni Lunâ Observatio Nova":

*a a a a a a a c c c c c d e e e e e e g h i i i i i i i*
*l l l l m m n n n n n n n n n n o o o o p p q r r s*
*t t t t t u u u u u*

which contains the letters of the following words: *Annulo cingitur, tenui, plano, nusquam cohaerente, ad eclipticam inclinato.*

An attack on this theory by Eustachio Divini followed shortly. This is published in full, together with Huygens' answer.

The volume contains further the observations of Saturn, made in Paris in 1668, through which Huygens corrects the value of the inclination of the plane of the ring to the ecliptic to about 31°; in 1668 the real value was 28° 12′.

A dozen shorter articles taken from the manuscripts follow, some of which are of considerable importance. These deal with a variety of subjects, of which his astronomical applications of the pendulum, his researches on the equation of time, and his instrument for measuring altitudes are the most important, and are good specimens of the brilliancy of Huygens' mind.

A. VAN MAANEN

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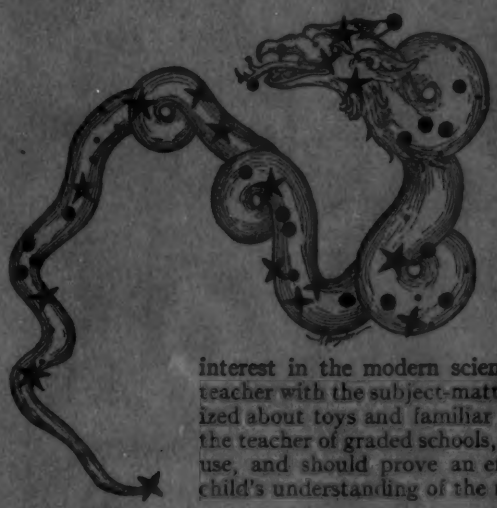

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